Decentralized Law Enforcement[*]

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(preliminary draft, comments are welcomed)

Abstract

The traditional approach taken in the literature on law enforcement focuses on the design of penalties and monitoring structures to manage undesirable behavior, assuming a single agent is in charge of designing and enforcing the law. However, for many violations enforcement is decentralized, carried out by many law enforcers who respond to incentives and take the law as given. This paper develops a theory of decentralized law enforcement in which a continuum of law enforcers, who have a negligible impact on the crime and arrest rates, chooses an enforcement level to apprehend law breakers. A continuum of citizens decides whether to break the law and the severity level of the offense. Assuming that law enforcers are rewarded every time they successfully apprehend a lawbreaker, I find that some policies aimed at discouraging law breaking can also reduce the level of enforcement, resulting in fewer citizens breaking the law, but each turning to more severe offenses. Also, lowering enforcement costs or increasing enforcement payoffs can in fact induce lower arrest rates. I show that announcing and committing to a given level of enforcement can lead to higher crime rates and lower apprehension rates, an effect that is aggravated when law enforcement is centralized. Finally, I consider how peer effects in law enforcement impact the levels of apprehension and crime, and also discuss some applications of the framework.

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1 Introduction

The traditional economic approach to law enforcement, pioneered by Becker (1968), sheds light on how a society *should* manage undesired behavior. Typically, it focuses on the design of punishment and monitoring structures to minimize social losses associated with crime. In principle, one can interpret this approach as one of *centralized* enforcement, in which there is one agent, such as a social planner, in charge of law enforcement. While this normative approach provides insightful foundations for law enforcement, it essentially dismisses the behavior of law enforcers who — like everyone else — respond to incentives and take the law as given. As Stigler (1970) points out: "...a law is not enforced by the society but by an agency that is instructed to do that task" (p. 531). To understand how enforcement is carried out, one needs to depart from the centralized or normative approach to crime.

This article examines the joint behavior of law enforcers and potential perpetrators. I analyze an entirely decentralized model that parallels the actual process in which enforcement is carried out. I think of enforcement as a two-sided interaction. Law enforcers are affected by the level of crime, whereas law breakers are concerned about the level of enforcement. Law enforcement is executed not by a single agent, but rather by a large number of law enforcement agents. Law enforcers choose a costly level of enforcement to capture offenders, who in turn decide whether to break the law and if so, choose the severity of the offense. The theory predicts a wide array of equilibrium observables: the level of enforcement, the offense severity, the detection probability, the crime rate, and the arrest rate.

Detecting and apprehending offenders is a task that yields an uncertain outcome. First, not all committed offenses are reported to the enforcers. According to the Bureau of Justice Statistics, around 3.4 million violent crimes annually went unreported to the police from 2006 to 2010. Thus, I assume that offenses are reported to law enforcers at given rate, which is formally equivalent to saying that law breakers randomly encounter law enforcers at that rate. Second, since not all crimes are cleared by arrest, enforcement is imperfect. According to the FBI, in 2012, 46.8% of violent crimes and 19.0% of property crimes were cleared by arrest or exceptional means.[1] I posit that the *detection chance* or the probability of capture rises with diminishing returns in both the level of enforcement and offense severity, which captures that law enforcers are more determined to look for more serious offenders.[2] Finally, I distinguish between detection chance and arrest rate. The crime rate is the mass of citizens who choose to break the law, while the arrest rate is the mass of those who are apprehended.

[1] Specifically, 22% of larceny-theft, 12.7% of burglary, and 11.9% of motor vehicle theft were cleared.

[2] As cited in (Quercioli and Smith, 2015, fn. 13): "If a counterfeiter goes out there and, you know, prints a million dollars, he's going to get caught right away because when you flood the market with that much fake currency, the Secret Service is going to be all over you very quickly." — Kersten (2005)

The detection chance is less than one, so the crime rate exceeds the arrest rate.

Enforcing and breaking the law are costly activities. Indeed, law enforcement expenditures in the US exceed 200 billion per year (Anderson, 2012). I assume that law enforcers make decisions at the margin, namely, each elects a costly *level of enforcement* (e.g. patrolling, monitoring, or policing). Critically, decentralization demands law enforcers to have negligible impact on the crime rate, or to take it as given. I assume that each law enforcer is rewarded every time she apprehends a law breaker, and so they effectively want to capture as many offenders as possible. Next, Becker (1968) supposes that a potential criminal can either commit a crime or not, but as Stigler (1970) points out: "marginal decisions are made here as in the remainder of life" (p. 527). I assume that each potential law breaker chooses whether to commit an offense, and if so, elects the *severity* of that offense.[3] A more severe offense yields a greater criminal (net) *reward*, but it faces a greater legal penalty if detected. I assume that potential offenders differ in their outside options (e.g., expected legal earnings); thus, only some of them will eventually choose to break the law.

I characterize the Nash equilibrium of this massive game, in which a continuum of law enforcers optimizes the level of enforcement anticipating the crime rate and the severity of the offenses, while a continuum of heterogeneous potential law breakers optimizes their entry and offense severity forecasting the enforcement level. The severity of the offense and the enforcement level fix the detection chance. Inspired by my work on crime with Lones Smith (Smith and Vásquez, 2016), an equilibrium can be partially seen as a competitive equilibrium of an "implicit market". Letting the respective enforcement level and the mass of crime to act as a market clearing price and quantity, a supply and demand framework emerges. The intersection of these curves yields a market equilibrium, which fixes the level of enforcement as a function of the offense severity. On the other hand, the optimization problem of law breakers yields a best response locus, fixing the offense severity as a function of the level of enforcement. In equilibrium, the offense severity is optimal given the level of enforcement, while the enforcement level clears the market given the offense severity. I prove that a unique equilibrium arises (Theorem 1),[4] affording unambiguous comparative statics.

I use the framework to study the equilibrium effects of harsher penalties, worse criminal outside options, better policing technology, greater enforcement costs, higher enforcement compensation, and greater criminal rewards. I next provide an overview of the predictions.

Changing the harshness of penalties is one of the key policy variables to deter undesired behavior. In Proposition 1, I consider two schemes of penalty enhancement. First, suppose

[3] For example, a motorist chooses how much to speed, or a thief how much to steal.

[4] Fixing the probability of capture, Burdett et al. (2003) introduce crime as an outside option into an otherwise standard labor search model. Their model exhibits multiple equilibria, as opposed to mine.

that penalties uniformly rise for all offenses. The profits of every law breaker fall, and so fewer potential offenders break the law. As a result, the crime rate falls. Thus, since each law enforcer faces a lower crime rate, the level of enforcement falls. This raises the marginal profitability of a more serious crime, incentivizing law breakers to commit more severe offenses.[5] With fewer criminals and each apprehended with lower chance, the arrest rate falls. Second, suppose that penalties are harsher the more severe the offense is. This effectively alters the marginal cost of an offense, incentivizing more citizens to choose their outside option, and those who break the law switch to less severe offenses. Still, the enforcement level, the detection chance, and the arrest rate fall, attenuating the drop in crime.

Proposition 1 finds a *substitution effect*: Higher penalties more than crowd out the apprehension probability. This implies that a policy maker cannot choose the detection chance and the penalties *independently*, as is typically assumed in the literature on public enforcement (Becker, 1968; Polinsky and Shavell, 2000).[6] When enforcement is decentralized this breaks down, for variations in penalties affect law breaking, which in turn affects enforcers' behavior.

The workhorse decision model of crime by Becker (1968) predicts that better employment opportunities — or outside options — discourage crime. This prediction emerges because of an implicit assumption that the level of enforcement does not respond to variations in the crime rate. Thus, improved outside options decrease the amount of law breaking and leave the offense severity unaffected — for a change in the extensive margin does not affect the intensive choice of an active law breaker. Nevertheless, Proposition 2 shows that once one accounts for the behavior of law enforcers, better outside options indeed raise the offense severity. The reason is that enforcers lower their enforcement level when fewer people break the law, ultimately increasing the marginal returns of a more severe offense. Altogether, there are fewer offenders, each breaking the law more intensively.[7]

Next, I turn to study the effectiveness of enforcement. Notably, when it becomes easier to apprehend criminals, the level of enforcement falls. Without accounting for feedback effects, one would have predicted the opposite. Since the probability of capture affects criminal behavior, Proposition 3 finds that greater law enforcing efficacy discourages both severe offenses and law breaking. These effects are strong enough to more than crowd out enforcement behavior, resulting in a lower detection chance and arrest rate.

[5] Focusing on the interaction between social norms and the enforcement of laws, Acemoglu and Jackson (2014) also find that higher penalties can have counteracting effects.

[6] Andreoni (1991) also finds that the probability of capture falls as penalties rise. In a model with fixed enforcement, a jury dictate the probability of conviction but need to be convinced "beyond a reasonable doubt". Malik (1990) argues that higher fines may induce criminal avoidance actions, lowering the capture chance.

[7] In a context of property crime, and so accounting for feedback effects between potential victims and criminals, Smith and Vásquez (2016) find that better outside options increase the number of attempted crimes per criminals, and could even increase the crime rate.

Proposition 4(a) then explores the equilibrium effects of greater enforcement costs. As apprehending perpetrators becomes more costly, law enforcers lower the level of enforcement. Thus, more people commit more severe crimes, since the profits and marginal profits of crime rise. The probability of capture falls. Yet, the arrest rate rises when the supply of crime is sufficiently elastic in enforcement level. Proposition 4(b) finds that the same qualitative predictions emerge as the enforcement payoff per criminal arrested falls.

Another way to discourage crime is by increasing the physical or technological costs of law breaking, or by lowering its rewards.[8] Proposition 5 explores how lower *net* criminal rewards impact crime and enforcement. I find that more people abide by the law and enforcers reduce their level of enforcement. The probability of capture and the arrest rate fall; however, the offense severity might rise depending on whether rewards are lower at the margin.

Next, I examine how different enforcement tactics or structures affect the levels of law breaking and apprehension. First, I explore the effects of announcing and sticking to a given level of enforcement. Essentially, I explore a variation of the game in which law enforcers move first. Proposition 6 shows that the new equilibrium entails a lower level of enforcement, and higher levels and intensities of law breaking. The logic is that in this scenario, law enforcers have a first-mover advantage, which they use for their private benefit. Since they are incentivized to arrest law breakers, in equilibrium they lower their enforcement level to raise the number of offenses. What is interesting is that the number of arrests does not necessarily go up: As long as the crime rate goes up, they can afford lower clearance rates.

Proposition 7 then studies an extreme case in which enforcement is completely centralized. This means that law enforcers fully cooperate and behave as a single unit, thus enjoy market power. Now enforcers can effectively impact the crime rate. I find that, given the incentive structure that enforcers face, centralization yields an unambiguously worse outcome relative to the case in which enforcement is announced. The bigger the market advantage that enforcers have, the worse the outcomes are. Surprisingly, announcing crackdowns or periods of intense policing might not be a good idea when enforcers are incentivized to catch offenders.

Finally, I study how peer effects impact the amount of enforcement and crime. When the enforcement behavior of others raises the marginal returns of an enforcer, the levels and intensities of law breaking fall. Yet, the level of enforcement might rise or fall depending on whether peer effects affect the efficacy or costs of enforcement (Corollaries 1–2).

Literature Review. I contribute to the literature on illegal behavior and public enforcement of laws. The classic reference is Becker (1968), who introduces the rational criminal and a supply of offenses that is affected by the probability of capture and the legal

[8] For instance, it is less profitable to commit a theft when potential victims are more vigilant; likewise, it is more costly to produce counterfeit money when genuine money is harder to imitate (Stigler, 1970).

penalty. He develops a normative analysis centered on how to design policies to minimize social losses, which triggered a vast literature on the optimal enforcement of laws.[9] In a nutshell, that literature studies the decision problem of a policy maker who chooses the probability of capture and the legal penalty to minimize social losses accounting for the optimal response of criminals. This approach essentially assumes not only perfect commitment, but also perfect coordination between law enforcers. While these assumptions make sense when there is a single agent in charge of enforcement, in reality this task is carried out by many law enforcers who respond to incentives and make choices at the margin. With many enforcers, each having negligible impact on aggregate variables, coordination is not trivial. I depart from this literature by explicitly modeling the behavior of a continuum of enforcers, and its interplay with potential law breakers. As a result, the detection chance, the crime rate and arrest rate all emerge as equilibrium outcomes of a large two-sided matching game.

Since law breakers make choices at the margin, this paper relates to the marginal deterrence literature (Stigler, 1970; Shavell, 1992; Mookherjee and Png, 1992, 1994). This literature also addresses normative issues, such as how policies should be designed to reduce the offense severity for the undeterred criminals.[10] None of these papers concentrate on the joint interaction between law enforcers and law breakers which is the main focus of this paper. I thus provide a vehicle to uncover the determinants of enforcement, apprehension and crime rates, and show not only how enforcement is carried out, but also how it is affected by different policies used to discourage undesired behavior.

This paper relates to the small literature that blends insights from matching and game theory to understand behavior in illegal markets. Smith and Vásquez (2016) center the analysis on theft markets, capturing the competitive nature of property crime as assets flow from victims to criminals. Quercioli and Smith (2015) develop a counterfeiting model as a multi-market game pitting counterfeiters against verifiers, and verifiers against each other, as fake money changes hands. While these models differ from mine in many dimensions (e.g., in both law enforcement is modeled as an "automaton"), they have the feature that agents stochastically encounter one another, and their actions are mutual best responses.[11]

Finally, Di Porto et al. (2013) develop a game-theoretic model of decentralized tax auditing in which law enforcers are rewarded for detecting criminals. The main focus of their paper is to take their model to the data and provide a method to calibrate it. They consider

[9] This literature is surveyed in, e.g., Polinsky and Shavell (2000).

[10] There are other related papers in which enforcement is centralized. Persico (2002) shows that constraining police to be fair does not necessarily reduce crime. Eeckhout et al. (2010) explain how announced random crackdowns can emerge as an optimal solution of police optimization. Lazear (2006) relates to this last point.

[11] There is less related literature that focuses on the interplay between laws and social norms, also applying insights from matching and game theory. See Acemoglu and Jackson (2014) and the references therein.

a fixed population of firms with heterogeneous true tax base (their type), which make a report to an auditor who cannot observe their type and chooses the auditing probability. In this paper, I assume that law enforcers observe whoever is breaking the law, and elect the enforcement level taking the detection technology as given. Like them, I assume that a law enforcer's payoff is affected by the level of crime — specifically, each enforcer is rewarded every time they apprehend a criminal.[12] This provides tractability and allows me to derive testable implications on enforcement behavior and crime, as each fundamental changes.

The rest of the paper proceeds as follows. In §2 I set up the model, and in §3 I provide a systematic analysis of the model. Next, in §4 I turn to explore implications of the model, contrasting my results with the literature. Finally, in §6 I discuss the effects of centralization and commitment as well as extensions of the framework. Section §7 concludes. Omitted proofs are in the Appendix.

2 The Model

I explore an anonymous equilibrium model in which all variables are measured in money.

A. Players. There is a unit measure of homogeneous risk neutral *law enforcers*, and a large measure of risk neutral heterogeneous *potential criminals*. Potential criminals differ in their *outside option* $\omega \geq 0$, which may represent expected legal earning opportunities. I assume that outside options are distributed according to an atomless distribution function $G(\omega)$ with density $g(\omega) \equiv dG(\omega)/d\omega > 0$ on $[0, \infty)$.

B. Actions and Random Matching. A law enforcer chooses a costly *enforcement level* $e \geq 0$ to capture criminals. One could think of e as the level of policing. A potential criminal chooses whether to commit a crime, and if so, he elects the *severity* $\sigma \geq 0$ of his offense.[13] Law enforcers and criminals are randomly matched in pairs. In particular, a law enforcer detects a law breaker with chance $\kappa \in [0, 1]$, the *crime rate*. The *arrest rate* $\alpha \in [0, 1]$ is the probability that a criminal is successfully apprehended.[14]

C. Stochastic Detection. As in Becker (1968), a potential criminal who commits a crime is then randomly caught. The enforcement level $e \geq 0$ and criminal severity $\sigma \geq 0$ fix the *detection chance* $\wp(e, \sigma) \in [0, 1]$, which is twice continuously differentiable.[15] Greater

[12] Di Porto et al. argue that this natural incentive scheme is not a bad idea for decentralizing incentives.

[13] There are numerous law breaking activities in which the intensive margin plays a key role. For instance, a pollutant decides how much to pollute, or a motor-vehicle driver chooses how much to speed, or a counterfeiter how much fake money to produce, or a drug-dealer how much drugs to carry, and so on. Even small offenses such as illegal parking involve a non-trivial intensive margin.

[14] The crime rate κ denotes the tightness of the market, namely the ratio of criminals to law enforcers. One can imagine that each law enforcer gets a new "criminal case" at rate κ, and "clears" it at rate α.

[15] One could also reinterpret the model and let κ to denote the detection rate, and $\wp$ the prosecution rate

police effort yields a higher detection chance, and so $\wp_e > 0$;[16] also, zero effort yields no detection, or $\wp(0, \sigma) = 0$ for any $\sigma \geq 0$. To secure in any equilibria a crime rate strictly greater than the arrest rate (and so an imperfect detection chance), I assume that for any severity σ, $\lim_{e \uparrow \infty} \wp(e, \sigma) \leq 1$. Next, to capture the idea that high severity crimes are less likely to go undetected, the detection chance $\wp$ weakly rises in the severity of the offense: $\wp_\sigma \geq 0$, with $\wp(e, 0) = 0$ for $e \geq 0$ if $\wp_\sigma > 0$, and $\lim_{\sigma \uparrow \infty} \wp(e, \sigma) \leq 1$ for $e > 0$. For instance, it should be easier to detect a driver who is way over the speed limit than one who is barely above. Next, as standard economic logic demands, effort and severity are subject to diminishing returns: $\wp_{ee}, \wp_{\sigma\sigma} \leq 0$, with $|\mathcal{E}_\sigma(\wp_\sigma)| \leq 1$. Any sufficiently concave function, such as the square-root $\sqrt{\cdot}$, satisfies this last inequality.

It is important to understand how the enforcement level e and the severity of the offense σ interact. I assume that high severity offenses weakly raise the marginal efficacy of enforcement, or $\wp_{e\sigma} \geq 0$. This encapsulates strategic interactions between enforcers and criminals. Also, I assume that complementarities are not too strong so that the detection chance $\wp$ is log-submodular in (e, σ) — such as multiplicative.[17,18] This means that the severity of an offense has relatively less effect in the detection chance when the enforcement is high. Any detection chance $\wp(e, \sigma) = e^a \sigma^b / [(1 + e^a)(1 + \sigma^b)]$ with $a, b \in (0, 1]$ obeys these assumptions.

D. A Law Breaker and Enforcer Optimization. A severity σ offense yields a net *reward* $r(\sigma)$ to the perpetrator, which is strictly increasing and concave: $r' > 0 \geq r''$.[19] To secure an interior optimal solution, I assume the Inada conditions that $\lim_{\sigma \uparrow \infty} r'(\sigma) = 0$ and $\lim_{\sigma \downarrow 0} r'(\sigma) = \infty$. Next, if a law breaker gets caught, then he needs to pay a *fine* $f(\sigma)$ that is strictly increasing and convex: $f', f'' > 0$.[20] Finally, the fine f is sufficiently convex so that a 1% increase in the severity of the offense raises the fine by at least 1% at the margin, or $\mathcal{E}_\sigma(f') \geq 1$. These assumptions hold for any geometric fine $f(\sigma) = \sigma^\phi$, with $\phi \geq 2$. All told, given an enforcement level e, a law breaker with outside option ω chooses the severity σ of

as in Mookherjee and Png (1994). Under this convention, the model could be easily adapted to shed light on how imperfections in the legal system affect enforcement and law breaking.

[16] For any smooth real valued function $x \mapsto h$ on $\mathbb{R}^n$, I define $h_{x_i}(x) \equiv \partial h(x)/\partial x_i$.

[17] A real valued function $x \mapsto h$ on a lattice $X \subseteq \mathbb{R}^n$ is *supermodular (submodular)* if $h(\max\{x, x'\}) + h(\min\{x, x'\}) \geq (\leq) h(x) + h(x')$. When h is twice differentiable, then h is supermodular (submodular) iff $h_{x_i x_j}(x) \geq (\leq) 0$ for all $i \neq j$, by Topkis (1978). These definitions are strict if the inequalities are strict. A positive function $h > 0$ is log-supermodular (log-submodular) if $\log(h)$ is supermodular (submodular).

[18] If $\wp$ is log-submodular, then $\log(\wp)_{e\sigma} \leq 0$ or $(\wp_\sigma/\wp)_e \leq 0$, which is equivalent to $\wp_{e\sigma} \leq \wp_\sigma \wp_e / \wp$.

[19] This function $r(\cdot)$ embeds the physical or technological costs of executing an offense. For property crime, the criminal rewards are negatively affected by victims' vigilant behavior. While I abstract from the conflict between victims and criminals, §4.5 explores the effects of lower criminal rewards. See Smith and Vásquez (2016) for a systematic analysis of theft markets, where the rivalry between criminals and victims is modeled.

[20] Shavell (1992) and Mookherjee and Png (1992) provide a foundation for why and when legal penalties should increase in the severity of the offense.

his offense to maximize *law breaking profits*:

$$\Pi(\sigma, e|\omega) \equiv r(\sigma) - \wp(e, \sigma)f(\sigma) - \omega \tag{1}$$

Thus, a citizen breaks the law when $\max_\sigma \Pi(\sigma, e|\omega) \geq 0$.

Now I turn to an enforcer's optimization. I assume that each law enforcer is risk neutral and behaves competitively, i.e. each has a negligible impact on the crime rate and the arrest rate. The cost of enforcement e is $c(e)$ which is twice continuously differentiable, increasing, and convex: $c', c'' > 0$ for $e > 0$, $c(0) = c'(0) = 0$, and $\lim_{e\uparrow\infty} c(e) = \infty$. Faced with a crime rate $\kappa > 0$ of severity σ crimes, an enforcer maximizes the arrest rate minus enforcement costs:[21]

$$V(e, \sigma, \kappa) \equiv \kappa\wp(e, \sigma)B - c(e) \tag{2}$$

where $B > 0$ is a constant *enforcement payoff* per criminal apprehended. Thereby, I assume that the enforcement level e is not contractible so that if enforcers' reward was not conditional of arrests, then they would shirk and no arrests would emerge. Observe how the decentralization of enforcement plays a role in an enforcer's objective. Since each enforcer take the crime rate — or the amount of crime — as given, each is incentivized to clear as many offenses as possible. This task elicits effort which is unobservable and costly.

E. Equilibrium. An *equilibrium* is a 5-tuple $(e^*, \sigma^*, \bar{\omega}^*, \kappa^*, \alpha^*)$ such that:

(i) Given the enforcement e^*, the severity σ^* maximizes criminal profits (1).

(ii) Given the severity σ^* and the crime rate κ^*, the enforcement e^* maximizes (2).

(iii) Given (σ^*, e^*), the marginal criminal $\bar{\omega}^*$ makes zero profits, i.e. $\Pi(\sigma^*, e^*|\bar{\omega}^*) = 0$. Only potential criminals with outside options $\omega \leq \bar{\omega}^*$ commit crimes.

(iv) Given $(e^*, \sigma^*, \bar{\omega}^*)$, the crime rate is $\kappa^* = G(\bar{\omega}^*)$, and the arrest rate $\alpha^* = \wp(e^*, \sigma^*)\kappa^*$.

An equilibrium here is a Nash equilibrium where potential criminals and enforcers optimize independently and simultaneously. Also, it could be seen as a long-run equilibrium where all margins, extensive and intensive, are fully flexible. But one could imagine that some margins should be fixed or "sticky" in the short-run. Typically, economists think of the extensive margin as the one which is fixed in the short-run. In this case, this means that the amount of law breaking is unchanged in the short-run, and thus one can view the crime rate κ is fixed in this run. In §4 I will explore how exogenous changes affect crime and enforcement behavior in both the short run and long run, in the same spirit of partial equilibrium analysis.

[21] In §6 I discuss how to adapt my model to allow for enforcement spillovers.

3 The Equilibrium Analysis

A. The Optimal Severity Locus. I now characterize the optimal intensive behavior of a criminal. Observe that for any police effort $e > 0$, the optimal severity is strictly positive since $\lim_{\sigma\downarrow 0} \Pi_\sigma(\sigma, e) = \infty$; and also, it is finite because $\lim_{\sigma\uparrow\infty} \Pi_\sigma(\sigma, e) < 0$. Thus, any optimal severity $\sigma \in (0, \infty)$ for a criminal who enters obeys:

$$\Pi_\sigma(\sigma, e|\omega) = r'(\sigma) - \wp_\sigma(e, \sigma)f(\sigma) - \wp(e, \sigma)f'(\sigma) = 0 \tag{3}$$

At any interior maximum, the marginal expected penalty equals the marginal benefit of a crime. A criminal is aware that raising the severity of his offense not only raises the legal penalty if caught, but also the chance of being apprehended. Clearly, all potential criminals who enter choose the same level of severity for any outside option ω.

It is not hard to see that the criminal profit function $\Pi(\sigma, e|\omega)$ is strictly submodular in (e, σ) for any outside options ω. Indeed, differentiating Π_σ from (3) in enforcement level e yields:

$$\Pi_{\sigma e}(\sigma, e|\omega) = -\wp_{\sigma e}(e, \sigma)f(\sigma) - \wp_e(e, \sigma)f'(\sigma) < 0$$

Thus, by Topkis (1978), the optimal severity $\Sigma^*(e) \equiv \arg\max_{\sigma\geq 0} \Pi(\sigma, e|\omega)$ of an offense falls in enforcement level e. As seen in Figure 1, the *(optimal) severity locus* Σ^* slopes down in (σ, e)-space, and so greater enforcement induces lower severity crimes. In other words, enforcement e and criminal severity σ are *strategic substitutes for law breakers* (Bulow et al., 1985): Greater enforcement lowers the marginal profitability of high severity crimes. This raises ambiguity on the detection chance. For a greater enforcement increases the detection chance, but it lowers the optimal severity, which in turn reduces the detection probability. Lemma 1 shows that in fact the direct effect is stronger, as one would have expected.

Lemma 1 *The detection chance $\wp$ rises as the enforcement rises along the severity locus Σ^*.*

B. The Supply of Crime. Given enforcement $e \geq 0$, the *marginal criminal function* $\bar{\omega}(e) \geq 0$ obeys:

$$\bar{\omega}(e) \equiv \max_{\sigma\geq 0} r(\sigma) - \wp(e, \sigma)f(\sigma) \tag{4}$$

Since potential criminals with outside option $\omega \leq \bar{\omega}$ enter, the *supply of crime* $\mathcal{K}^S(e)$ is:

$$\mathcal{K}^S(e) \equiv G(\bar{\omega}(e)) \tag{5}$$

By the Envelope Theorem, the marginal criminal $\bar{\omega}(\cdot)$ falls in the level of enforcement, because $\bar{\omega}'(e) = -\wp_e(e,\sigma)f(\sigma) < 0$ with $\sigma = \Sigma^*(e)$ solving (3). As a result,

$$\frac{d\mathcal{K}^S(e)}{de} = g(\bar{\omega}(e))\bar{\omega}'(e) < 0 \tag{6}$$

So *the supply of crime $\mathcal{K}^S(e)$ falls in the enforcement level e.* Observe that the enforcement level e acts as a "price" in this implicit market for crime — for if the enforcement is low, then more people break the law, by (6). Since the market good is a bad, the "market supply of crime" slopes down. If criminals were homogeneous (degenerate G), the supply of crime would be perfectly inelastic at some enforcement level. When potential criminals are heterogeneous, the supply becomes less inelastic. Also, distributions that are more "disperse", in the sense of having a greater ratio g/G, make the supply less inelastic. For intuitively, a small change in the enforcement level affects a smaller set of potential perpetrators, and thus it has a lower impact on the supply of crime. Finally, the supply of crime is perfectly inelastic at some crime rate level in the short run, as seen in Figure 1.

A rise in the enforcement level lowers the supply of crime, by (6), but it rises the chance that a criminal is apprehended, by Lemma 1. A natural question then emerges: Does the arrest rate, or the total number of arrests, rises? Intuitively, when the enforcement is high, there are fewer crimes, and each is more likely to be cleared. How drastic enforcement impacts the total number of crimes is likely to be related to the outside option distribution G. Indeed, log-differentiate the arrest rate $\wp(e,\Sigma^*(e))\mathcal{K}^S(e)$ in the enforcement e:

$$\frac{d\log\wp\mathcal{K}^S}{de} = \frac{\wp_e}{\wp} + \frac{\wp_\sigma}{\wp}\frac{d\Sigma^*}{de} + \frac{g}{G}\bar{\omega}' \tag{7}$$

Observe that if g/G is small enough (i.e., $g/G \sim 0$), then the arrest rate rises in the enforcement, by Lemma 1. However, if g/G is sufficiently high (i.e., $g/G \sim \infty$), then the arrest rate falls in the enforcement level. The intuition is that, if the outside option distribution is sufficiently "disperse", then an increase in the level of enforcement has a minor effect on criminal exit. But, if the outside option distribution is too "concentrated", then as enforcement rises, criminals exit so much that even if the detection chance rises, the arrest rate falls.

Using that criminals make non-negative profits, and that each chooses the severity of their offense optimally, I find a marginal version of Becker's cost-benefit inequality.

Lemma 2 *At any optimum, $r'/r > \wp_\sigma/\wp + f'/f$.*

Proof: In any equilibrium $\bar{\omega} > 0$, i.e. $r > \wp f$ by (4). Thus, by FOC (3) and $r > \wp f$:

$$r' = \wp_\sigma f + \wp f' = \wp f\left(\frac{\wp_\sigma}{\wp} + \frac{f'}{f}\right) < r\left(\frac{\wp_\sigma}{\wp} + \frac{f'}{f}\right) \iff \frac{r'}{r} > \frac{\wp_\sigma}{\wp} + \frac{f'}{f} \qquad \square$$

So a citizen breaks the law only if the rewards are greater than the expected penalty, and the elasticity of the rewards exceed the sum of that of the fine and probability of capture.

C. The Derived Demand for Crime. Now let me turn to a law enforcer's optimization problem. Faced with a crime rate $\kappa > 0$ of severity $\sigma > 0$ crimes, a law enforcer maximizes (2). Since $c(e) \to \infty$ as $e \to \infty$, any optimum is finite. Likewise, any optimum is positive, for $c'(0) = 0$ and $\wp_e(e, \sigma) > 0$ as $e \downarrow 0$. Hence, at any optimum, enforcement effort is positive and finite, and thus obeys the first-order condition:

$$V_e(e, \sigma, \kappa) = \kappa \wp_e(e, \sigma) B - c'(e) = 0 \tag{8}$$

Note that the payoff function $V(\cdot)$ is supermodular in $(e, (\sigma, \kappa))$. Indeed,

$$V_{e\sigma}(e, \sigma, \kappa) = \kappa \wp_{e\sigma}(e, \sigma) B > 0 \qquad \text{and} \qquad V_{e\kappa}(e, \sigma, \kappa) = \wp_e(e, \sigma) B > 0$$

Hence, by Topkis (1978), the best-reply $\mathcal{E}(\sigma, \kappa) \equiv \arg\max_e V(e, \sigma, \kappa)$ rises in (σ, κ), in the sense that if $\sigma' \geq \sigma$ and $\kappa' \geq \kappa$, then $\mathcal{E}(\sigma', \kappa') \geq \mathcal{E}(\sigma, \kappa)$. So, unlike criminals, for a fixed crime rate κ, effort e and severity σ are *strategic complements for law enforcers.*

Now, fix severity σ and solve for κ in (8) to get a *derived demand for crime* $\mathcal{K}^D(e|\sigma) = \kappa$, where

$$\mathcal{K}^D(e|\sigma) \equiv c'(e)/[\wp_e(e, \sigma) B] \tag{9}$$

Observe that, unlike the supply of crime $\mathcal{K}^S$, the derived demand $\mathcal{K}^D(e|\sigma)$ depends on the criminal severity σ. Also, as seen in Figure 1, *the derive demand* $\mathcal{K}^D(\cdot|\sigma)$ slopes up in the enforcement level e and shifts left in the severity of the offense σ.

Notice that if the severity did not affect the detection chance, namely, $\wp_\sigma = 0$, then the the supply and demand framework would encapsulate all the interaction between enforcers and criminals. For the demand (9) would not depend on the severity of the offense.

D. The Market Clearing Locus. For any severity σ, the enforcement level e that clears the market obeys $\mathcal{K}^S(e) = \mathcal{K}^D(e|\sigma)$. This generates the *market clearing locus* $\mathcal{C}^*$ in the (e, σ)-space as in Smith and Vásquez (2016). This locus is well defined by Lemma 3 below. Note that a rise in σ shifts demand $\mathcal{K}^D$ left, and so the market clearing enforcement rises along $\mathcal{K}^S$ (middle panel of Figure 1). Therefore, in (σ, e)-space the $\mathcal{C}^*$ locus slopes up.

Lemma 3 *For all severity $\sigma > 0$, there exists a unique effort $e^* \in (0, \infty)$ that clears this induced market for crime, or $\mathcal{K}^D(e^*, \sigma) = \mathcal{K}^S(e^*)$.*

Observe that the behavior of all agents is effectively projected into a graphical framework in the space of severity and enforcement levels (σ, e). In any equilibrium, the criminal

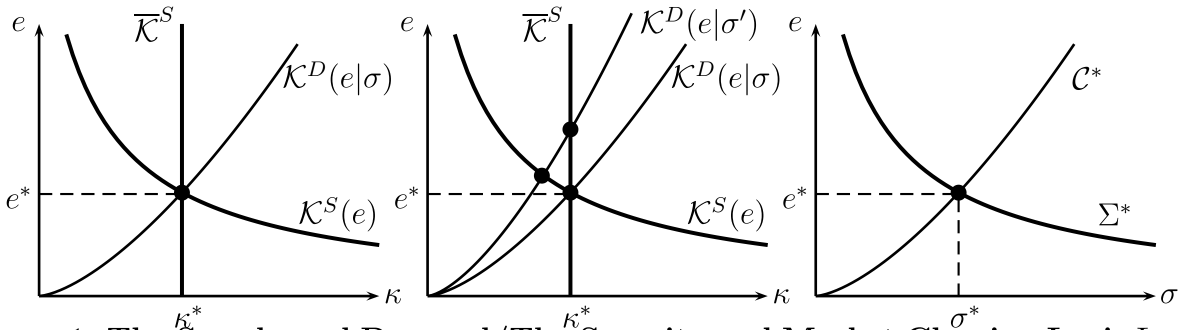

Figure 1: **The Supply and Demand/The Severity and Market Clearing Loci.** LEFT: The supply $\mathcal{K}^S$ slopes down in e in the long-run, and is perfectly inelastic in the short-run; while the demand $\mathcal{K}^D$ slopes up in e. MIDDLE: A rise in the severity from σ to σ' shifts the demand up and left. The enforcement rises more in the short-run and it attenuates in the long-run, as the crime rate falls. RIGHT: The respective market clearing $\mathcal{C}^*$ and severity Σ^* loci slope up and down in (σ, e)-space.

severity σ^* must be a best-reply to the enforcement e^*, and the enforcement e^* must clear the market given the severity σ^*. In other words, any intersection of Σ^* and $\mathcal{C}^*$ generates an equilibrium. By the slopes of the optimal severity Σ^* and market clearing $\mathcal{C}^*$ loci, there is a unique intersection (σ^*, e^*), which argues the existence and uniqueness of crime equilibrium. Let $\xi(\kappa) \equiv (G(\bar{\omega}))^{-1}(\kappa)$ denotes the *inverse supply of crime*.

Theorem 1 *Suppose that* $c'(\xi(1))/\wp_e(\xi(1), \Sigma(\xi(1))) < B$. *Then, there exists a unique crime equilibrium* $(e^*, \sigma^*, \kappa^*, \alpha^*)$. *In equilibrium, the crime rate* $\kappa^* < 1$.

Proof: Consider the optimal severity locus. By the Implicit Function Theorem (IFT), the FOC (3) yields a smooth best-reply map $e \mapsto \Sigma^*$. Since $\wp(0, \sigma) = 0$, $\Sigma^*(e) \uparrow \infty$ as $e \downarrow 0$. Also, $\Sigma^*(e) \downarrow \underline{\sigma} \equiv \arg\max_\sigma r(\sigma) - f(\sigma)$ as $e \uparrow \infty$, for $\wp(e, \sigma) \uparrow 1$ as $e \uparrow \infty$. Now by the Inverse Function Theorem, write $e \mapsto \Sigma^*$ inversely as $\sigma \mapsto e_{\Sigma^*}$. Thus, e_{Σ^*} obeys: $e_{\Sigma^*}(\sigma) \downarrow 0$ as $\sigma \uparrow \infty$, and $e_{\Sigma^*}(\sigma) \uparrow \infty$ as $\sigma \downarrow \underline{\sigma}$. Now the market clearing locus. By IFT again, this locus is generated by a smooth map $\sigma \mapsto e_{\mathcal{C}^*}$. Notice that $e_{\mathcal{C}^*}(\underline{\sigma}) > 0$, and $e_{\mathcal{C}^*}(\sigma) \uparrow \bar{e} > 0$ as $\sigma \uparrow \infty$, by Lemma 3. Now define $\Delta e \equiv e_{\Sigma^*} - e_{\mathcal{C}^*}$. By our previous analysis, $\Delta e(\sigma) \uparrow \infty$ as $\sigma \downarrow \underline{\sigma}$, and $\Delta e(\sigma) \downarrow -\bar{e}$ as $\sigma \uparrow \infty$. Altogether, by the Intermediate Value Theorem, there exists σ^* with $\Delta e(\sigma^*) = 0$. Next, since Δe is monotone decreasing (for $\Delta e'(\sigma) = e'_{\Sigma^*}(\sigma) - e'_{\mathcal{C}^*}(\sigma) < 0$), it follows that σ^* is unique. Finally, define $e^* = e_{\Sigma^*}(\sigma^*) = e_{\mathcal{C}^*}(\sigma^*)$, $\bar{\omega}^* = \bar{\omega}(e^*)$, $\kappa^* = G(\bar{\omega}^*)$, and $\alpha = \wp(e^*, \sigma^*)\kappa^*$. Altogether, $(e^*, \sigma^*, \bar{\omega}^*, \kappa^*, \alpha^*)$ constitutes an equilibrium.

Now let me show that $\kappa^* < 1$. In equilibrium, $\kappa^* = G(\bar{\omega}(e^*)) \leq 1$ iff $e^* \geq \xi(1)$. Take $e = \xi(1)$ and $\sigma = \Sigma(\xi(1))$. I'll show that (e, σ) induces an excess of supply. Indeed, if $e = \xi(1)$ then $\mathcal{K}^S(e) = 1$; while the demand $\mathcal{K}^D(e|\sigma) = c'(\xi(1))/[B\wp_e(\xi(1), \Sigma(\xi(1)))] < 1 = \mathcal{K}^S(e)$, by assumption. So the market clearing enforcement must rise, or $e^* \geq \xi(1)$, and so $\kappa^* \leq 1$. □

4 Testable Implications

4.1 Harsher Punishment

In this section, I explore the effects of harsher penalties. To this end, smoothly index the fine $f(\sigma|\varphi)$ so that $f_\varphi(\sigma|\varphi) > 0$, where $\varphi \in \mathbb{R}$. When the fine $f(\sigma|\varphi)$ is log-supermodular in (σ, φ), a rise in φ raises the elasticity of the fine in the offense severity σ.[22] I refer to a greater φ as *harsher marginal punishments*. One could also imagine another policy that raises the fine $f(\sigma|\varphi)$ uniformly for all severity levels (e.g. speeding tickets become more expensive independently of the actual stop speed) so that wlog $f(\sigma|\varphi) \equiv f(\sigma) + \varphi$.[23] In this case, I refer to an increase in φ as *harsher uniform penalties*.

Next, I show that harsher marginal punishments lower the level of enforcement, the amount of law breaking, and the severity of the offenses conditional on law breaking. However, harsher uniform penalties might have counteracting effects, discouraging law breaking, but incentivizing more severe offenses conditional on law breaking.

Proposition 1 (Harsher Punishment) *(a) If penalties are marginally harsher, then the enforcement level e, the offense severity σ, and the arrest rate fall. If the marginal efficacy of enforcement $\wp_e$ is slightly affected by the offense severity, then the crime rate κ falls. (b) If penalties are uniformly harsher, then the enforcement level e, the arrest rate, and the crime rate fall. The offense severity level rises if g/G is high enough and $\wp$ is log-modular, and falls if g/G is low enough.*

Proof part (a): First, since $f_\varphi > 0$ and f log-supermodular in (σ, φ), the fine f is supermodular in (σ, φ), or $f'_\varphi > 0$, by Topkis (1998). Next, twice differentiating (1) yields $\Pi_{\varphi\sigma} = -\wp_\sigma(e, \sigma) f_\varphi(\sigma|\varphi) - \wp(e, \sigma) f'_\varphi(\sigma|\varphi)$. Thus, Π is submodular in (σ, φ), since $f'_\varphi > 0$. So By Topkis (1998), the optimal severity locus $\Sigma^*(e|\varphi) \in \arg\max_\sigma \Pi(\sigma, e|\varphi)$ shifts left, as seen in the middle panel of Figure 2.

Next, I turn to (κ, e)-space. By the Envelope Theorem in (4), $\bar{\omega}_\varphi = -\wp(e, \sigma) f_\varphi(\sigma|\varphi) < 0$, where $\sigma = \Sigma^*(e)$. Hence, the supply of crime $\mathcal{K}^S(e|\varphi)$ shifts left in φ. Since the demand for crime $\mathcal{K}^D$ is constant in φ, the market clearing enforcement falls along the demand curve (see the left panel of Figure 2); thus, the market clearing locus $\mathcal{C}^*$ shifts down. Altogether, the equilibrium enforcement level e unambiguously falls.

[22]For instance, this property holds if φ raises the convexity of the fine f in the sense of Arrow-Pratt. Indeed, a rise in φ is associated with a more convex fine iff the marginal fine $f'(\sigma|\varphi)$ is log-supermodular (Diamond and Stiglitz, 1974). But then, this implies that the fine $f(\sigma|\varphi)$ is log-supermodular in (σ, φ). For let $\mathbb{I}(\sigma, \sigma') \equiv 1$ if $\sigma' \leq \sigma$ and 0 otherwise. Since $\mathbb{I}(\sigma, \sigma')$ is log-supermodular in (σ, σ'), the function $\mathbb{I}(\cdot) f'(\cdot)$ is log-supermodular in $(\sigma, \sigma', \varphi)$; and thus, $f(\sigma|\varphi) = \int_0^\infty \mathbb{I}(\sigma, \sigma') f'(\sigma'|\varphi) d\sigma'$ is log-supermodular in (σ, φ), since log-supermodularity is preserved under integration by Karlin and Rinot (1980).

[23]Since $f'_\varphi = 0$, the fine $f(\sigma|\tilde{\varphi}) \equiv f(\sigma) + h(\tilde{\varphi})$. If h is not linear, then one could define $\varphi \equiv h(\tilde{\varphi})$.

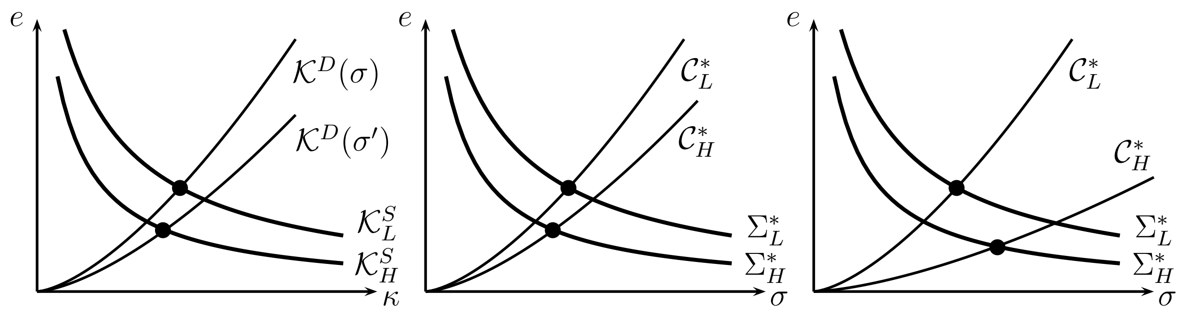

Figure 2: **A Harsher Punishment Lowers the Enforcement and the Crime rate.** When the fine rises from L to H, the supply shifts left from $\mathcal{K}_L^S$ to $\mathcal{K}_H^S$. The market clearing $\mathcal{C}^*$ and the severity Σ^* loci shift from $\mathcal{C}_L$ to $\mathcal{C}_H$ and from Σ_L^* to Σ_H^*, respectively. The enforcement and the severity fall. A parallel shift of the fine implies less enforcement and greater severity.

Next, observe that the equilibrium severity σ falls if the $\mathcal{C}^*$ locus shifts down less than Σ^*, which is the case here. Indeed, fix σ and log-differentiate the market clearing $\mathcal{C}^*$ locus $\mathcal{K}^D(e,\sigma)\equiv\mathcal{K}^S(e|\varphi)$, and the severity Σ^* locus $r'(\sigma)\equiv\wp_\sigma(e,\sigma)f(\sigma|\varphi)-\wp(e,\sigma)f'(\sigma|\varphi)$ to get:

$$\left.\frac{de}{d\varphi}\right|_{\mathcal{C}^*}\left(\frac{c''}{c'}-\frac{\wp_{ee}}{\wp_e}-\frac{g}{G}\bar{\omega}'\right)=\frac{g}{G}\bar{\omega}_\varphi \quad \text{and} \quad \left.\frac{de}{d\varphi}\right|_{\Sigma^*}\left(\frac{\wp_{\sigma e}}{\wp_e}+\frac{f'}{f}\right)=-\frac{\wp f_\varphi}{\wp_e f}\left(\frac{\wp_\sigma}{\wp}+\frac{f'_\varphi}{f_\varphi}\right) \tag{10}$$

Since $c''>0>\wp_{ee}$, the slope $de/d\varphi|_{\mathcal{C}^*}>-(g/G)\bar{\omega}_\varphi/[(g/G)\bar{\omega}']=-\bar{\omega}_\varphi/\bar{\omega}'=-\wp f_\varphi/(\wp_e f)$. Thus, $de/d\varphi|_{\mathcal{C}^*}>-\wp f_\varphi/(\wp_e f)$. By looking at $de/d\varphi|_{\Sigma^*}$ in (10), a sufficient condition to ensure that the equilibrium severity falls is:

$$\frac{\wp_\sigma}{\wp}+\frac{f'_\varphi}{f_\varphi}\geq\frac{\wp_{\sigma e}}{\wp_e}+\frac{f'}{f} \iff \frac{f'_\varphi}{f_\varphi}-\frac{f'}{f}\geq\frac{\wp_{\sigma e}}{\wp_e}-\frac{\wp_\sigma}{\wp} \tag{11}$$

For then $de/d\varphi|_{\Sigma^*}\leq-\wp f_\varphi/(\wp_e f)<de/d\varphi|_{\mathcal{C}^*}$. Notice that (11) holds, since $\wp(e,\sigma)$ is log-submodular — and so the right side of the right expression in (11) is negative — and $f(\sigma|\varphi)$ is log-supermodular — and so the left side of the right expression in (11) is positive.

Now, let us explore changes in the arrest rate. Slightly abusing notation, let $\alpha(e,\sigma)\equiv\wp(e,\sigma)\mathcal{K}^D(e,\sigma)=c'(e)\wp(e,\sigma)/[\wp_e(e,\sigma|\varphi)B]$. Note that α rises in e (for $\wp_{ee}<0<c''$) and in σ (for $\wp_e/\wp$ falls in σ). Thus, $\alpha(e(\varphi),\sigma(\varphi))$ falls as φ rises, since $e'(\varphi),\sigma'(\varphi)<0$.

Finally, the crime rate. Since $\kappa=\mathcal{K}^S(e|\varphi)$, a rise in φ induces a positive indirect effect on the crime rate: A greater φ lowers enforcement, and thereby it raises the crime rate. So the net effect is ambiguous. Yet, if the marginal efficacy of enforcement $\wp_e$ is slightly affected by the offense severity σ, then the crime rate falls. For suppose that $\wp_{e\sigma}=0$. Then the demand curve $\mathcal{K}^D$ in (9) is unaffected by the offense severity, and thus, the demand $\mathcal{K}^D$ falls in φ, since the enforcement level falls in φ. As a result, the crime rate $\kappa=\mathcal{K}^D(e|\sigma)$ falls in φ. By continuity, this arguments obtains when $\wp_{e\sigma}$ is small enough. □

Proof of part (b): By the same logic of part (a), the enforcement level unambiguously falls, for the market clearing $\mathcal{C}^*$ and severity Σ^* loci shift down. The effect on the offense severity is more subtle. For suppose that $g/G \sim \infty$ and that $\wp$ is log-modular, then $de/d\varphi|_{\mathcal{C}^*} = -\bar{\omega}_\varphi/\bar{\omega}' = -\wp f_\varphi/(\wp_e f)$ by (10). But then, looking at $de/d\varphi|_{\Sigma^*}$ in (10), we deduce that condition (11) fails since $f'_\varphi = 0$ and $\wp_{e\sigma}/\wp_e = \wp_\sigma/\wp$. Altogether, the offense severity rises. Next, the crime rate falls, since the demand $\mathcal{K}^D(e|\sigma)$ falls as e falls and σ rises. Finally, the arrest rate. By Lemma 1, one can deduce that the level curves of the detection chance $\wp(e, \sigma)$ are more elastic than Σ^*. Thus, the probability of capture $\wp$ falls, given the right panel of Figure 2. Since the probability of capture and the crime rate fall, the arrest rate falls.

Now let $g/G \sim 0$ as in the short-run. Then the clearing locus $\mathcal{C}^*$ is unaffected, while the severity locus Σ^* shifts left. So the enforcement and offense severity unambiguously fall. The crime rate is unchange, while the capture probability falls. Thus, the arrest rate falls.□

Proposition 1 finds that harsher penalties discourage law breaking behavior, as one would expected. That harsher penalties reduce crime rate has been documented in, e.g., Hansen (2015) in the context of driving under the influence, and Helland and Tabarrok (2007) in the context of California's three strikes (for excellent surveys; see Durlauf and Nagin (2010); Chalfin and McCrary (2014)). Proposition 1, however, distinguishes two types of harsher penalties, for one of them has stronger marginal deterrent effects than the other. Specifically, raising marginal penalties not only has a deterrent effect but also a marginal deterrent effect, as opposed to harsher uniform penalties. What is interesting is that uniform penalties may raise the intensity of law breaking only because enforcers lower their behavior, which underscores the importance of feedback effects on law breaking behavior.

4.2 Better Outside Options

Smoothly index the outside option distribution $G(\omega|\varphi)$, where $\varphi \in \mathbb{R}$. Say that *outside options are better* if $G(\cdot|\varphi_H) < G(\cdot|\varphi_L)$ when $\varphi_L < \varphi_H$. That is, $G(\cdot|\varphi_H)$ is better than $G(\cdot|\varphi_L)$ in the sense of First-Order Stochastic Dominance. One can think about better outside options as a stronger labor market, or lower unemployment, or higher wages, and so on. Alternatively, it could denote the profits of a competing criminal alternative.

I find that when outside options are better, criminals commit more severe crimes, and law enforcers exert less effort. The crime and arrest rate fall. So I have fewer criminal, each committing more severe offenses and getting caught at lower rates.

Proposition 2 (Better Outside Options) *If outside options are better, then enforcement, the crime rate, and arrest rate all fall. The offense severity rises.*

Proof: First, the supply of crime $\mathcal{K}^S(e|\varphi) \equiv G(\bar{\omega}(e)|\varphi)$ falls in φ for any enforcement e. Next, since the demand for crime $\mathcal{K}^D$ is unaffected, the market clearing enforcement falls

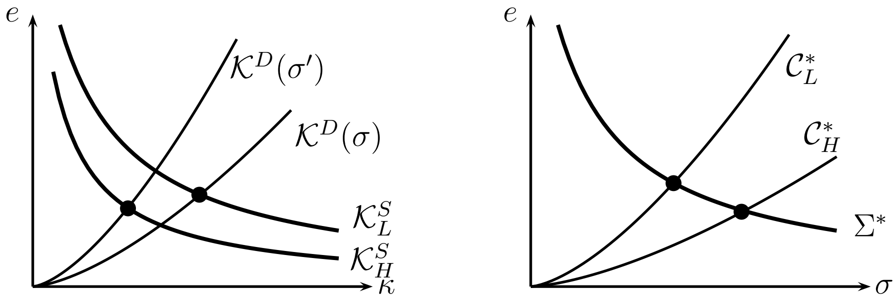

Figure 3: **Better Outside Options.** When the outside option distribution shifts from L to H, the supply shifts left from $\mathcal{K}^S_L$ to $\mathcal{K}^S_H$, lowering the crime rate and the enforcement level. The market clearing locus shifts down to $\mathcal{C}^*_H$ from $\mathcal{C}^*_L$. The severity rises to $\sigma' > \sigma$, and the crime rate falls even more, as the demand shifts to $\mathcal{K}^D(\sigma')$ from $\mathcal{K}^D(\sigma)$.

along the demand $\mathcal{K}^D$ — as depicted in Figure 3. As a result, the market clearing locus $\mathcal{C}^*$ shifts down in (σ, e)-space. Since the optimal severity locus Σ^* is constant in φ, by (3), I have that the enforcement e^* falls, but criminal severity σ^* rises (right panel of Figure 3).

Now the crime and arrest rates. Since demand $\mathcal{K}^D$ shifts left in σ (see §3-C), the crime rate falls due to two reinforcing effects — namely, supply $\mathcal{K}(\cdot|\varphi)$ and demand $\mathcal{K}^D(\cdot, \sigma)$ shift left (see Figure 3). Finally, the arrest rate $\alpha = \wp(e, \sigma)\mathcal{K}^D(e, \sigma)$ falls, since the detection chance $\wp$ falls a e falls along Σ^* (Lemma 1), and the crime rate falls. $\square$

Proposition 2 underscores the importance of feedback effects on crime to rationalize empirical findings. Focusing solely on the criminal side, worse outside options would incentives entry without affecting the severity of the offenses or the enforcement level. Indeed, following the classic decision approach to crime (Becker, 1968), one would expect less offenses of the same severity, and no variation in the level of enforcement. However, once one accounts for feedback effects, I find that *the severity of each offense rises whereas the enforcement level falls*. The reason is that the supply of crime shifts left, while the demand is unchanged; thus, the level of enforcement falls and the severity rises — as seen in the right panel of Figure 2. Nevertheless, the crime rate and arrest rate fall. Altogether, there are fewer citizens breaking the law, but each breaking it more severely.

Reversing the logic, Proposition 2 finds that worse outside options raise the crime rate but reduce the severity of the crimes. For example, Bignon et al. (2015) documents the evolution of crime rates in France in 19th century. From 1826-1936 the phylloxera crises destroyed 40% of France's vineyards — which can be viewed a negative income shock, or worse outside options — and caused a substantial increase in property crime and a significant decrease in violent crime. Proposition 2 could make sense to this empirical finding, provided that violent crimes are more severe than property crimes.

4.3 Changing the Detection Technology

I now explore how the equilibrium changes when the detection technology improves. To this end, suppose the detection chance is indexed by $\varphi \in \mathbb{R}$ so that $\wp(e, \sigma|\varphi)$. When $\wp(e, \sigma|\varphi)$ is log-supermodular in (e, φ) and modular in (σ, φ), a greater φ raises the relative marginal efficacy of police (i.e. $\wp_e/\wp$ rises), but leaves invariant the relative rate of change $\wp_\sigma/\wp$. I call a greater φ with $\wp_\varphi > 0$ an *increase in enforcement efficacy.*

Proposition 3 (Detection Technology) *If enforcement efficacy rises, then the offense severity σ, and arrest rate α all fall. The level of enforcement rises in the short-run, but it falls in the long-run. If $\wp$ is log-modular in (e, σ), then the crime rate κ fall.*

Proof: First, consider the (κ, e)-space. The demand for crime $\mathcal{K}^D(e, \sigma|\varphi) = c'(e)/[\wp_e(e, \sigma|\varphi)B]$ shifts left, because $\wp$ is log-supermodular in (e, φ) and increasing in φ, and so supermodular in (e, φ) (i.e. $\wp_e$ rises in φ). The supply of crime is unaffected in the short-run, for the marginal criminal is fixed. So the (short-run) market clearing enforcement rises.

In the long-run, or with flexible entry, the supply of crime $\mathcal{K}^S(e|\varphi) = G(\bar{\omega}(e|\varphi))$ shifts left, since $\mathcal{K}^S_\varphi = g\bar{\omega}_\varphi = -g\wp_\varphi f < 0$, by the Envelope Theorem in (4). Thus, the crime rate falls at any severity; however, the effect on the market clearing enforcement is ambiguous. It falls (rises) if the demand shifts left more (less) than the supply does. Fix $e > 0$, and differentiate in φ:

$$\frac{1}{\mathcal{K}^S}\frac{\partial \mathcal{K}^S}{\partial \varphi} - \frac{1}{\mathcal{K}^D}\frac{\partial \mathcal{K}^D}{\partial \varphi} = -\frac{g}{G}\wp_\varphi f + \frac{\wp_{e\varphi}}{\wp_e} \tag{12}$$

Next, the severity locus $\Sigma^*(e, \varphi) = \arg\max_\sigma \Pi(\sigma, e|\varphi)$ shifts left, since criminal profit $\Pi(\cdot)$ is submodular in (σ, φ). Indeed, twice differentiating $\Pi(\cdot)$ yields $\Pi_{\sigma\varphi} = -(\wp_{\sigma\varphi} f + \wp_\varphi f) < 0$, since $\wp_\varphi > 0$, and $\wp_{\sigma\varphi} > 0$ by log-modularity of $\wp$ in (σ, φ). Now, I consider two cases.

Case 1: $\mathcal{C}^*$ shifts right. Here, $\mathcal{K}^S_\varphi - \mathcal{K}^D_\varphi < 0$, or $\wp_{e\varphi}/\wp < (g/G)\wp_\varphi f$ by (12). Thus, as seen in Figure 4, the equilibrium enforcement e^* falls. The effect on the equilibrium severity σ^* is more subtle. By Claim A.1, the Σ^* locus shifts down more than $\mathcal{C}^*$ does, and so the offense severity σ^* falls.

Case 2: $\mathcal{C}^*$ shifts left. Here, $\wp_{e\varphi}/\wp > (g/G)\wp_\varphi f$, or equivalently, $\mathcal{E}_e(\wp_\varphi) > |\mathcal{E}_e(\mathcal{K}^S)|$. So $\mathcal{K}^S_\varphi - \mathcal{K}^D_\varphi > 0$ by (12). As seen in Figure 4, the severity σ^* falls. The effect on the enforcement e^* is more complicated, since the shifts of $\mathcal{C}^*$ and Σ^* push the equilibrium enforcement in opposite directions. Claim A.2 shows that the equilibrium effort e^* falls, since the Σ^* locus shifts left more than $\mathcal{C}^*$.

Now the crime rate. This comparative static is hard, because on one hand a higher φ lowers the enforcement level e, and so it induces more criminal entry; but on the other, a

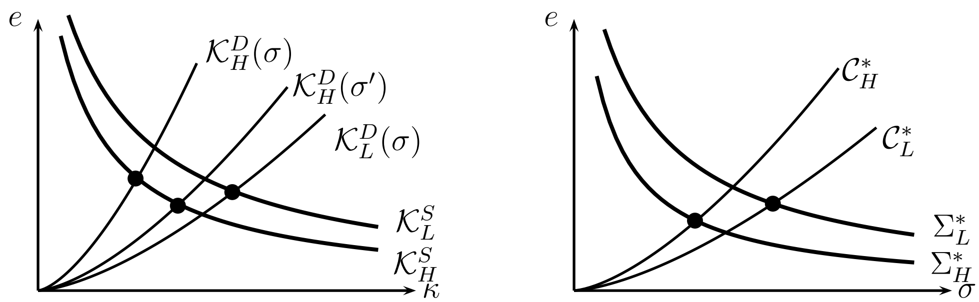

Figure 4: **Better Enforcement Efficacy Lowers the Enforcement Level: Proposition 3 depicted.** If police efficacy rises from L to H, then the supply and demand shift left from $\mathcal{K}_L^S$ to $\mathcal{K}_H^S$ and from $\mathcal{K}_L^D$ to $\mathcal{K}_H^D$, respectively. If demand shifts more than supply, then the market clearing enforcement rises, and the market clearing locus shifts up to $\mathcal{C}_H^*$ from $\mathcal{C}_L^*$. The optimal severity locus shifts left to Σ_H^* from Σ_L^*, lowering the equilibrium enforcement and the offense severity.

higher φ raises the detection chance, discouraging entry. Appendix A.3 shows that the latter force wins out, and thus the crime rate falls.

Finally, the arrest rate. Slightly abusing notation, let $\alpha(e,\sigma,\varphi) \equiv \wp(e,\sigma|\varphi)\mathcal{K}^D(e,\sigma|\varphi) = c'(e)\wp(e,\sigma|\varphi)/[\wp_e(e,\sigma|\varphi)B]$. Note that α rises in e (for $\wp_{ee} < 0 < c''$) and in σ (for $\wp_e/\wp$ falls in σ), but it falls in φ (for $\wp_e/\wp$ rises in φ). Altogether, $\alpha(e(\varphi),\sigma(\varphi),\varphi)$ falls as φ rises. $\square$

Notably, Proposition 3 suggests that improving the efficacy of enforcement could paradoxically reduce the level of enforcement. The critical reason is that law enforcers respond to variations in the crime rate. Since potential law breakers anticipate a greater detection chance, few of them break the law and each turn to minor offenses. Even though, an increase in enforcement efficacy incentives enforcers to increase their behavior, since the crime rate and offense severity fall, this latter indirect effects dominates, lowering the level of enforcement. Nevertheless, in the short-run — when the crime rate is fixed — a greater enforcement efficacy increase the level of enforcement, as predicted by decision models of crime.

Now let me discuss some applications of the theory. First, while I do not explicitly account for an spatial dimension, the theory could distinguish between urban and rural areas — for intuitively, it is hard to capture criminals in big cities given their substantial density. Assuming that the marginal efficacy of enforcement is lower in cities than in rural areas, Proposition 3 predicts more severe offenses and higher crime and arrest rates in cities than in rural areas — as documented in Glaeser and Sacerdote (1999). For an alternative application, DeAngelo and Hansen (2014) show that a higher number of traffic accident in Oregon emerged as a result of a substantial cut of state troopers, consistent with Proposition 3. Finally, Proposition 3 also makes sense to the empirical finding that "Hot-Spot Policing" or crackdowns — i.e., or periods with high volume of police interventions in specific areas — cause reductions in crime (see McCrary and Chalfin (2011) and references therein).

4.4 Greater Enforcement Costs or Payoffs

In this section I examine the equilibrium effects of lower enforcement costs or payoffs. To this end, smoothly parametrize the enforcement cost $c(e|\varphi)$, where $\varphi \in \mathbb{R}$. If enforcement costs of $c(e|\varphi)$ are log-submodular in (e, φ), then a greater φ lowers the elasticity of c in the enforcement level e. Say that *enforcement costs fall* if φ rises and $c_\varphi < 0$.[24]

Proposition 4 (Lower Enforcement Costs or Greater Enforcement Payoffs) *If enforcement costs falls, then the enforcement level rises, while the offense severity and crime rate fall. The arrest rate rises if g/G is sufficiently small; but it falls if g/G is high enough. The same predictions obtain when the enforcement payoff B rises.*

Proof: First, in the (κ, e)-space, the demand for crime $\mathcal{K}^D(e, \sigma|\varphi) = c'(e|\varphi)/[\wp_e(e, \sigma)B]$ shifts left, since $c'_\varphi(e|\varphi) < 0$ by log-submodularity of $c(\cdot)$ and $c_\varphi < 0$. The supply of crime $\mathcal{K}^S(\cdot)$ in (5) is unaffected. Thus, the market clearing enforcement shifts up along the supply curve — as depicted in Figure 5. So in the (σ, e)-space, the market clearing locus $\mathcal{C}^*$ shifts left and up. The optimal severity locus $\Sigma^*(e) = \arg\max_\sigma \Pi(\sigma, e|\omega)$ is unchanged. Hence, as seen in Figure 5, the level of enforcement e^* rises and the offense severity σ^* falls.

Next, the crime rate falls since by (5), $d\mathcal{K}^s/d\varphi = g\bar{\omega}'(e)de/d\varphi > 0$ since $\bar{\omega}' < 0 < de/d\varphi$. The effect on the arrest rate is more subtle, since the detection chance $\wp$ rises in the enforcement along Σ^* (Lemma 1), the detection chance rises in φ. So in the short-run — when the crime rate is fixed — the arrest rate rises. In the long-run, if g/G is small enough, then a change in e induced by φ has a minor effect on the crime rate, and thus the arrest rate rises. But, if $g/G \sim \infty$ then an increase in the enforcement level induces excessive compliance which depresses the crime rate falls so much, that the arrest rate rises even if the probability of capture $\wp$ falls (see discussion in §3-A). The Appendix shows that a greater enforcement payoff B yields the same qualitative predictions. □

Proposition 4 suggests that lower enforcement costs could in fact be associated with lower arrest rates. The reason is that lower marginal enforcement costs unambiguously raise the behavior of each enforcer, reducing profits and marginal profits of an offense. Thus, potential law breakers respond with fewer and less severe offenses, depressing the crime rate so much that in the end only fewer law breakers are arrested, even though the probability of capture raises. Notably, if one had not accounted for feedback effects, then one would have expected more arrests, predictions that emerge in the model in the short-run.[25]

[24] By the same logic of §4.1, this property holds if a rise in φ lowers the convexity of c in the sense of Arrow-Pratt, or equally well, if marginal costs $c'(e|\varphi)$ are log-submodular in (e, φ).

[25] Jin and Lee (2014) using restaurant hygiene inspection data from 2003-2009 in the state of Florida find that the adoption of PDA (portable digital assistant) led enforcers to detect more violations (greater arrest rate) and induce more compliance from restaurants (lower offense severity) — consistent with Proposition 4.

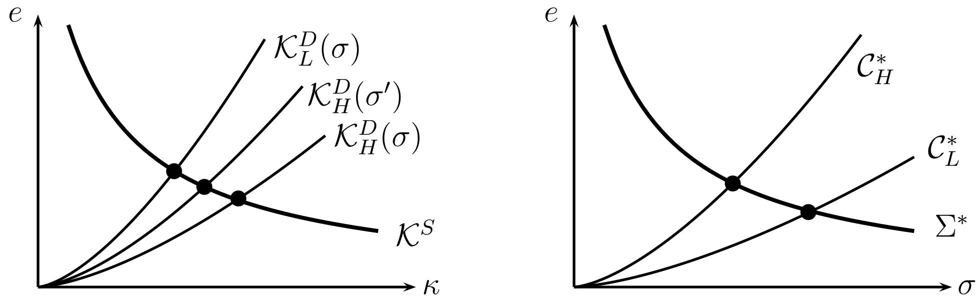

Figure 5: **Higher Enforcement Costs or Lower Enforcement Payoff.** When enforcement costs rise from L to H, the demand shifts right from $\mathcal{K}_L^D$ to $\mathcal{K}_H^D$. Given an offense severity, the market clearing enforcement and crime rate rise. The market clearing locus falls from $\mathcal{C}_L^*$ to $\mathcal{C}^*$, and the optimal severity locus Σ^* is unchanged. The offense severity rises, attenuating the rise of the crime rate.

Finally, observe that lowering enforcement costs uniformly — i.e., without affecting its marginal costs — has no impact the the equilibrium outcomes. The reason is that enforcers are not effectively incentivized at the margin, and so the shock does not propagate in the market. Consistent with this, Garicano and Heaton (2010) using police data show that an increase in information technology (IT) alone are not associated with reductions in crime rates, or increases in arrests rates — for intuitively, IT lowers the fix costs of enforcement.

4.5 Higher Criminal Rewards

Now I explore the consequences of higher criminal rewards. Assume that criminal rewards are smoothly parametrized so that $r(\sigma|\varphi)$, where $\varphi \in \mathbb{R}$, with $r_\varphi(\cdot|\varphi) > 0$. Assume that $r(\cdot)$ is log-supermodular in (σ, φ) so that the relative marginal rewards r'/r rises in φ. So *higher marginal rewards* φ raises the proportional marginal returns of a more severe offenses. On the other hand, if r is modular in (σ, φ), then *higher uniform rewards* φ increase law breaking rewards by exactly the same amount. I secure different predictions for each shift, because of the feedback effects that govern this market.

Proposition 5 (Higher Criminal Rewards) *(a) If rewards are marginally higher, then the enforcement level, the offense severity, and the arrest rate rise. If the detection chance is slightly affected by the offense severity, then the crime rate rises. (b) If rewards are uniformly higher, the enforcement level, arrest rate, and crime rate rise; the offense severity falls.*

Proof of part (a): First, in the (κ, e)-space, a rise in φ shifts the supply of crime $\mathcal{K}^S(e|\varphi) = G(\bar{\omega}(e|\varphi))$ in (5) to the right, for by the Envelope Theorem: $\bar{\omega}_\varphi(e|\varphi) = r_\varphi(\sigma|\varphi) > 0$ with $\sigma = \Sigma^*(e)$. The demand for crime $\mathcal{K}^D(e, \sigma)$ in (9) is clearly unaffected. Thus, the market clearing enforcement e^* rises at any severity σ (see Figure 6).

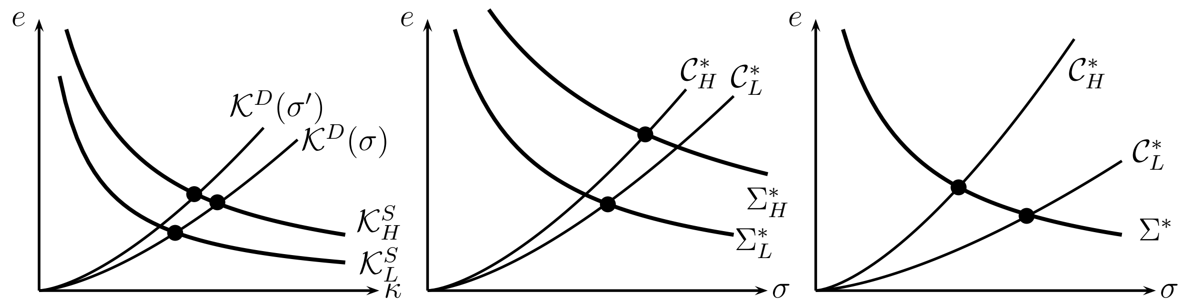

Figure 6: **Greater Criminal Rewards: Proposition 5 depicted.** When criminal rewards marginally rise from L to H, supply shifts right form $\mathcal{K}^S_L$ to $\mathcal{K}^S_H$. The market clearing enforcement and crime rate rise for a given offense severity. The optima severity locus shifts right from Σ^*_L to Σ^*_H, whereas the market clearing locus shifts left from $\mathcal{C}^*_L$ to $\mathcal{C}^*_H$. The equilibrium enforcement level rises, and so does the offense severity, since Σ^* shifts up more than $\mathcal{C}^*$ does. When rewards uniformly rise, surprisingly the severity level falls (right panel).

Second, in the (σ, e)-space, the market clearing locus $\mathcal{C}^*$ shifts left and up, by step 1. Next, criminal profits Π are supermodular in (σ, φ), for $\Pi_{\sigma\varphi} = r'_\varphi > 0$, given log-supermodularity of r and $r_\varphi > 0$. Altogether, the optimal severity locus $\Sigma^*(e, \varphi) = \arg\max_\sigma \Pi(\sigma, e|\varphi)$ shifts right. As seen in Figure 6, the equilibrium effort e^* clearly rises. But, the equilibrium severity σ^* might rise or fall depending on the magnitude of the shifts. I find that the Σ^* locus shifts up more than $\mathcal{C}^*$ does, and thus σ^* rises — as depicted in Figure 6. Indeed, Fix σ and differentiate (3), or $\Pi_\sigma(\sigma, e|\varphi) \equiv 0$, and log-differentiate $\mathcal{K}(e, \sigma)^D \equiv \mathcal{K}^S(e, \varphi)$ in φ to get:

$$\left.\frac{de}{d\varphi}\right|_{\mathcal{C}^*} \left(\frac{c''}{c'} - \frac{\wp_{ee}}{\wp_e} + \frac{g}{G}\wp_e f\right) = \frac{g}{G} r_\varphi \quad \text{and} \quad \left.\frac{de}{d\varphi}\right|_{\Sigma^*} \left(\frac{\wp_{\sigma e}}{\wp_e} + \frac{f'}{f}\right) = \frac{r'_\varphi}{\wp_e f} \tag{13}$$

Now clearly $de/d\varphi|_{\mathcal{C}^*} < r_\varphi/(\wp_e f)$, since $c''/c' > 0 > \wp_{ee}/\wp_e$. If $\wp$ is log-submodular in (e, σ), then given that r is log-supermodular in (σ, φ) and Lemma 2:

$$\frac{\wp_{\sigma e}}{\wp_e} + \frac{f'}{f} \leq \frac{\wp_\sigma}{\wp} + \frac{f'}{f} < \frac{r'}{r} \leq \frac{r'_\varphi}{r_\varphi} \tag{14}$$

Use (14) to bound $de/d\varphi|_{\Sigma^*}$ in (13) and get: $de/d\varphi|_{\Sigma^*} \geq r_\varphi/(\wp_e f) > de/d\varphi|_{\mathcal{C}^*}$. So σ^* rises.

Finally, the arrest rate $\alpha(e, \sigma) \equiv \wp(e, \sigma)\mathcal{K}^D(e, \sigma) = (\wp/\wp_e)(c'/B)$ rises too, since $\alpha_e > 0$ (for $c'' > 0 > \wp_{ee}$), and $\alpha_\sigma > 0$ (for $\wp/\wp_e$ rises in σ by log-submodularity of $\wp$). Thus, the arrest rate $\alpha(e(\varphi), \sigma(\varphi))$ rises in φ, for $e'(\varphi), \sigma'(\varphi) > 0$. Now the crime rate. If the detection chance slightly respond to the offense severity, or $\wp_\sigma \sim 0$, then the crime rate κ^* rises, for $d\kappa/d\varphi = d\mathcal{K}^D(e(\varphi), \sigma(\varphi))/d\varphi \sim \mathcal{K}^D_e de/d\varphi > 0$, given (9) and $e'(\varphi) > 0$.

Proof of Part (b): Follows directly from the proof of Proposition 2. □

When rewards are marginally higher, then more citizens break the law, and each turn to more serious offenses. Law enforcers respond with a greater enforcement level raising the probability of capture, and thus *attenuating* law breaker's marginal incentives. All told, law breaking is more serious and happens at higher rates, but at the same time, more law breakers are apprehended. In this case, feedback effects undermine law breaking incentives totally and at the margin. Surprisingly, when rewards rise uniformly, *law breakers in fact switch to less serious crimes*. Feedback effects are so strong that they more than crowd out the law breakers' incentives. The logic is that greater uniform rewards do not alter the marginal incentives of perpetrators, but rather encourage more people to break the law. Law enforcers raise the enforcement level lowering the marginal profitability of a serious offense. A decision model of crime would have predicted more offenses of the same severity.

Next, I discuss how Proposition 5 sheds light on some important economic applications. First, suppose that the offense in question is drug dealing, so that σ measures a quantity of drugs. Furthermore, suppose that rewards represent revenues, so that $r(\sigma) = \Delta\sigma$, where Δ is the unit price of drugs. The model provides neat insights on what happens when the price of drugs d rise due to, say, an increase in the demand for drugs. We should expect more people smuggling greater amounts of drugs, each of them getting caught at higher rates. Second, consider investment on security to prevent theft. One could imagine that such investments impose a fixed costs on criminal opportunities.[26] Reversing the logic of Proposition 5, the theory predicts fewer people breaking the law, but each breaking it more intensively and getting arrested a lower rates. Next, consider counterfeiting. Fixing the denomination, suppose that genuine bills are harder to imitate, imposing a fix cost on counterfeiting. Then the theory predicts fewer counterfeiters, each counterfeiting more money, which is the market structure that casual empiricism suggests. Finally, Proposition 5 provides novel insights on taxation in which a central question is to determine how increases in tax rates affect tax evasion. For in my paper, greater tax rates raises the marginal rewards of tax evasion, and so Proposition 5 not only predicts more tax evasion — as documented in Fisman and Wei (2004) — but also greater levels of enforcement and apprehension.

5 Announcing or Centralizing the Enforcement Level

5.1 The Effects of Announcing the Enforcement Level

In this section, I apply the framework to study the effects of "announced" law enforcement, as discussed in, e.g., Lazear (2006) and Eeckhout et al. (2010). These papers explore the effectiveness of announcing strategies in a context of centralized deterrence, namely, when enforcers behave a single agent affecting directly the crime rate. I ask the same question but

[26] See Smith and Vásquez (2016) and Draca and Machin (2015) and their references therein.

now examining the polar case when enforcement is totally decentralized and enforcers care about arrests.

Suppose that every law enforcer i commits to an enforcement level e_i. One could think of different submarkets where the level of enforcement is e_i in submarket i. Potential law breakers choose in which submarket to break the law. Since law enforcers are homogeneous, all of them choose the same enforcement level in equilibrium, say e. Thus, law breakers choose offenses of severity $\Sigma^*(e)$, given (3). Hence, in the representative submarket, the level of enforcement must solve:

$$\max_{e\geq 0} \kappa\wp(e, \Sigma^*(e))B - c(e)$$

Observe that the crime rate κ is not affected, since enforcers are atomless. The FOC yields:

$$\kappa \left(\wp_e(e, \Sigma^*) + \wp_\sigma(e, \Sigma^*)\frac{d\Sigma^*}{de} \right) B - c'(e) = 0 \tag{15}$$

Notice that the parenthesized term is positive by Lemma 1. Also, the second expression in the parenthesized term captures the effect of commitment. While this effect does not impact the supply of crime $\mathcal{K}^S$ in (5), it does affect the demand curve. Solving for κ in (15) yields an updated demand curve $\hat{\mathcal{K}}^D(e)$, that unlike the demand in (9), this one depends only the enforcement level,

$$\hat{\mathcal{K}}^D(e) = \frac{c'(e)/B}{\wp_e(e, \Sigma^*) + \wp_\sigma(e, \Sigma^*)\frac{d\Sigma^*}{de}}$$

An equilibrium in this setting is precisely the intersection point between $\mathcal{K}^S$ in (5) and the demand $\hat{\mathcal{K}}^D$ above. Next, I show how announcing the level of enforcement affect crime and enforcement. To be precise, call $(\hat{e}, \hat{\sigma}, \hat{\kappa}, \hat{\alpha})$ the equilibrium in which the enforcement level is announced, and $(e^*, \sigma^*, \kappa^*, \alpha^*)$ the original equilibrium (where the choices are simultaneous).

Proposition 6 *The new equilibrium obeys $\hat{e} < e^*$, $\hat{\kappa} > \kappa^*$, $\hat{\sigma} > \sigma$. The arrest rate can either rise or fall depending on the supply elasticity, or the ratio g/G.*

Proof: I'll show that the equilibrium enforcement is lower when enforcement is announced, or $\hat{e} < e^*$. Suppose that the crime rate is κ^*. Since $\Sigma^*(e^*) = \sigma^*$ the left side of (15) at $e = e^*$ obeys:

$$\kappa^* \left(\wp_e(e^*, \sigma^*) + \wp_\sigma(e, \sigma^*)\frac{d\Sigma^*}{de} \right) B - c'(e^*) = \kappa^*\wp_\sigma(e^*, \sigma^*)\frac{d\Sigma^*}{de}B < 0$$

Since by definition $e^* = \arg\max_{e\geq 0} \kappa^*\wp(e, \sigma^*)B - c(e)$, and $d\Sigma^*/de < 0 < \wp_\sigma$. Hence, the demand curve $\hat{\mathcal{K}}^D$ shifts down relative to $\mathcal{K}^D$ in (9), as seen in Figure 7. Since the supply curve $\mathcal{K}^S$ in (5) is unaffected, in the new equilibrium the enforcement level $\hat{e} < e^*$ and the crime rate $\hat{\kappa} > \kappa^*$. The severity rises since $\hat{\sigma} = \Sigma^*(\hat{e}) > \Sigma^*(e^*) = \sigma^*$. Finally, when the

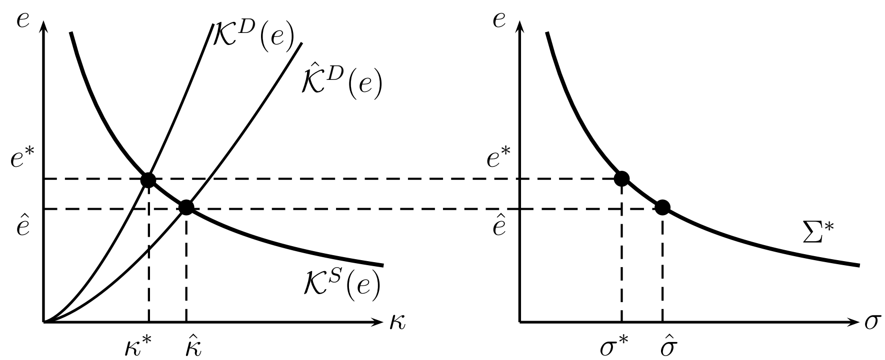

Figure 7: **Announcing Enforcement Strategies Raises the Crime Rate: Proposition 6 Depicted.** LEFT: The supply $\mathcal{K}^S$ is unaffected by whether law enforcement is announced or not, while the demand $\hat{\mathcal{K}}^D$ is lower than $\mathcal{K}^D$ when enforcement is not announced. The equilibrium enforcement falls and crime rate rises. RIGHT: The severity rises from σ^* to $\hat{\sigma}$ as the enforcement level falls from e^* to $\hat{e}$.

supply is sufficiently elastic in that g/G is high, then $\hat{\alpha} > \alpha^*$. However, if the supply is not too elastic, or g/G is low, then $\hat{\alpha} < \alpha^*$ (see discussion after (7)). □

Surprisingly, announcing the enforcement level raises the crime rate and the severity of the offenses, leading to a worse outcome.

5.2 Centralized Enforcement and Market Power

In this section I explore how enforcement and crime are affected by full-cooperative enforcement behavior. This assumption implicitly says that one could think that law enforcers are "robots" who obey the orders coming from a single authority. What happens now is that level of enforcement will directly affect the crime rate, resembling a monopolist exerting market power. Essentially, law enforcers instead of being crime rate takers, now they are crime rate makers.[27] Let me keep the market story, and suppose that the central enforcement unit charges a price e. Then each law breaker demands a quantity of the good $\sigma = \Sigma^*(e)$, by (3). As a result, slightly abusing notation, the aggregate demand that the monopolist faces is $\kappa(e) \equiv \mathcal{K}^S(e) = G(\bar{\omega}(e))$. All told, the monopolist chooses the price e to maximize profits, or to solve:

$$\max_{e \geq 0} \kappa(e)\wp(e, \Sigma^*(e))B - c(e)$$

An interior optimum necessarily solves the first order condition below:

$$\kappa(e)\left(\wp_e(e, \Sigma^*) + \wp_\sigma(e, \Sigma^*)\frac{d\Sigma^*}{de}\right)B + \kappa'(e)\wp(e, \Sigma^*)B - c'(e) = 0$$

[27] I thank Marek Weretka for suggesting me this analogy.

Note that unlike (15), the FOC has an extra term $(\kappa'(e)\wp(e,\Sigma^*)B)$ which captures the effects of market power on enforcement determination. Call $(\tilde{e},\tilde{\sigma},\tilde{\kappa},\tilde{\alpha})$ the equilibrium in which the enforcement is centralized, $(\hat{e},\hat{\sigma},\hat{\kappa},\hat{\alpha})$ the same in which enforcement is decentralized but announced, and $(e^*,\sigma^*,\kappa^*,\alpha^*)$ when behavior is decentralized. Comparing the equilibrium outcomes of these three different market structures yields a striking result.

Proposition 7 *The centralized equilibrium obeys* $\tilde{e}<\hat{e}<e^*$, $\tilde{\sigma}>\hat{\sigma}>\sigma^*$, *and* $\tilde{\kappa}>\hat{\kappa}>\kappa^*$.

Proof: First, $\tilde{e}<\hat{e}$. For since $\hat{e}$ solves (15) and $\hat{\kappa}=\kappa(\hat{e})$, $\tilde{e}=\hat{e}$ cannot be an optimum:

$$\hat{\kappa}\left(\wp_e(\hat{e},\Sigma^*)+\wp_\sigma(\hat{e},\Sigma^*)\frac{d\Sigma^*}{de}\right)B+\kappa'(\hat{e})\wp(\hat{e},\Sigma^*)B-c'(\hat{e})=\kappa'(\hat{e})\wp(\hat{e},\Sigma^*)B<0$$

Therefore, $\tilde{e}<\hat{e}$ for the marginal profits fall in enforcement e. Next, the offense severity obeys $\tilde{\sigma}=\Sigma^*(\tilde{e})>\Sigma^*(\hat{e})=\hat{e}$, given (3); and the crime rate $\tilde{\kappa}=G(\omega(\tilde{e}))>G(\omega(\hat{e}))=\hat{\kappa}$, given (4). Finally, the rest of the inequalities fall from Proposition 6. $\square$

When enforcement is centralized and enforcers are compensated every time they apprehend a law breaker, they coordinate their behavior to induce a sufficiently high crime rate in order to raise the number of apprehended law breakers. So decentralizing incentives yields an unambiguously better outcome. Thus, when enforcement is centralized, compensation should not be conditional on arrests — for instead of coordinating to achieve lower crime rates and greater clearance rates, exactly the opposite happens.

6 Applications and Extensions of the Framework

6.1 Spillovers of Law Enforcement

In this section I show how to adapt the model to capture social interactions in law enforcement, so that law enforcers care about the behavior of other enforcers. Let me consider that law enforcers are affected by their own actions and the action of others, captured by the *average* enforcement level $\bar{e}$. Next, consider a detection chance $\hat{\wp}(e,\sigma,\bar{e})$.[28] To fix ideas, suppose that $\hat{\wp}_e(e,\sigma,\bar{e})$ rises in $\bar{e}$ so that there are positive spillovers in law enforcement. Given a crime rate κ, offense severity σ, and average enforcement level $\bar{e}$, an enforcer solves

$$\max_{e\geq 0}\kappa\hat{\wp}(e,\sigma,\bar{e})B-c(e)$$

[28]For instance, if $\hat{\wp}(e,\sigma,\bar{e})\equiv e\wp(\bar{e},\sigma)$ then a enforcer has a negligible apprehending criminals, for variations in her enforcement level has no effect on the detection chance $\wp$. See Persico (2009) for a related discussion.

Observe that fixing the crime rate κ and the offense severity σ, enforcers face a none trivial interaction between one another. In fact, it is not hard to see that this game is supermodular, for the marginal payoff of each enforcer rises as the average level of enforcement rises. This games typically exhibit multiple equilibria (Vives, 1990; Milgrom and Roberts, 1990). Yet, since in my model law enforcers are homogeneous, in equilibrium all choose the enforcement level, say e. Thus, in equilibrium, e solves the above optimization at $\bar{e} = e$:

$$\kappa\hat{\wp}_e(e,\sigma,\bar{e})B - c'(e)|_{\bar{e}=e} = 0 \quad \Longrightarrow \quad \mathcal{K}^D(e|\sigma) = \frac{c'(e)}{\hat{\wp}_e(e,\sigma,e)B}$$

As in §3, this optimization yields a derived demand curve. Thus, looking at (9), all the results of this paper carry over to this setting, provided $\wp_e(e,\sigma) \equiv \hat{\wp}_e(e,\sigma,e)$. Call $(\hat{e},\hat{\sigma},\hat{\kappa},\hat{\alpha})$ the equilibrium in which the probability of capture is $\hat{\wp}$, and $(e^*,\sigma^*,\kappa^*,\alpha^*)$ the same in which the probability of capture is $\wp$. Presumably, fixing the individual enforcement level e and the offense severity σ, the apprehension chance $\hat{\wp}(e,\sigma,\bar{e}) > \wp(e,\sigma)$ for all $\bar{e}$, due to peer effects.

Corollary 1 *Suppose that $\hat{\wp}_e(e,\sigma,e)/\hat{\wp} > \wp_e(e,\sigma)/\wp$ and $\hat{\wp}_\sigma(e,\sigma,e)/\hat{\wp} = \wp_\sigma(e,\sigma)/\wp$ for all e,σ.*[29] *Then $\hat{e} < e^*$, $\hat{\sigma} < \sigma^*$, $\hat{\alpha} < \alpha^*$. If $\wp$ and $\hat{\wp}$ are log-modular in (e,σ), then $\hat{\kappa} < \kappa^*$.*

Proof: Direct from Proposition 3. □

Intuitively, if there are positive spillovers in law enforcement that improve the marginal efficacy of an enforcer, then in equilibrium one should observe fewer arrests, a not only a lower crime rate, but also less severe offenses. But spillovers could also arise from "team work", in the sense that the marginal cost of a single enforcer falls in the enforcement level of others, or $\hat{c}_{e\bar{e}}(e,\bar{e}) < 0$. In this case the enforcement optimization turns to

$$\max_{e\geq 0} \kappa\wp(e,\sigma)B - \hat{c}(e,\bar{e})$$

Again, this optimization is supermodular, but since law enforcers are homogeneous, in equilibrium all choose the enforcement level, say e. Thus, the optimal e obeys:

$$\kappa\wp_e(e,\sigma)B - \hat{c}_e(e,\bar{e})|_{\bar{e}=e} = 0 \quad \Longrightarrow \quad \mathcal{K}^D(e|\sigma) = \frac{\hat{c}_e(e,e)}{\wp_e(e,\sigma)B}$$

An immediate application of Proposition 4 sheds light on how team work affects crime.

Corollary 2 *Suppose that $\hat{c}_e(e,e)/\hat{c} < c_e(e)/c$ for all e. Then $\hat{e} > e^*$, $\hat{\sigma} < \sigma^*$, $\hat{\kappa} < \kappa^*$.*

Proof: Direct from Proposition 4. □

[29] For instance, this assumption holds when $\hat{\wp} = e\wp(\bar{e},\sigma)$ (Persico, 2009) with $\wp$ multiplicative in (e,σ).

6.2 Avoiding Law Enforcement: An Application to Drug Dealing

It is not hard to imagine that law breakers might expend resources avoiding being caught. This is intuitively true for drug dealing and other kinds of smuggling. Passing 100 pounds of drugs from one country to another elicits more efforts to avoid being notice than trying to pass 1 pound. Now I explore how to extend my model to speak to this specific application.

Suppose drug smugglers can incur in a costly action $a \geq 0$ to reduce the chance of being apprehended, so that the detection chance is now $\hat{\wp}(e, \sigma, a)$ with $\wp_a < 0$. I can think of the severity of the offense σ as the quantity of drugs produced. As standard economic logic demands, a drug dealer can reduce the detection chance at decreasing rates (i.e., $\hat{\wp}_{aa} > 0$); also, for simplicity, assume that avoidance costs are linear. Thus, given an enforcement level e, a criminal chooses how much to smuggle σ and how much to expend avoiding enforcers a to maximize:[30]

$$\hat{\Pi}(\sigma, a, e|\omega) \equiv r(\sigma) - \hat{\wp}(e, a, \sigma) f(\sigma) - a - \omega$$

Next, consider the following maximization in stages. First, a drug dealer chooses how much to deal σ, and *then* decides how to avoid law enforcers a. So in the second stage, a drug dealer chooses $a^*(e, \sigma) \equiv \arg\max_{a \geq 0} \hat{\Pi}(\sigma, a, e)$, given enforcement e and quantity σ. my earlier results thus would carry over to this setting, provided $\wp(e, \sigma) \equiv \hat{\wp}(e, a^*(e, \sigma), \sigma)$ and $\Pi(\sigma, e) \equiv \max_{a \geq 0} \hat{\Pi}(\sigma, a, e)$. However, to understand how changes in the environment affect avoidance efforts a would substantially complicate the comparative statics, for there is a nontrivial interactive effect between the offender's intensive margins.

7 Concluding Remarks

For many, if not all, violations, the enforcement of laws is executed by many law enforcers who — like everyone else — respond to incentives. While this seems natural, the traditional approach in the literature is silent on this dimension. In this paper I developed a decentralized theory of enforcement focusing on the interplay between law enforcers and law breakers. Enforcement behavior is affected by the level of crime, while criminal behavior is influenced by the enforcement level. Law breakers and enforcers randomly encounter and in equilibrium their actions are mutual best responses. I provide a general and tractable framework for the

[30] Becker et al. (2006) analyze a model of drug dealing and centralized enforcement, in which (1) drug dealers produce drugs according to a constant returns to scale technology, (2) the amount of drug produces has no effect on the detection chance, and (3) the penalty rises linearly in the quantity of drugs. In my model this translates to having linear net rewards $r''(\sigma) = 0$, a detection chance with $\wp_\sigma = 0$ and a fine with $f'' = 0$, trivializing the intensive margin σ. In a context of centralized enforcement, Malik (1990) focuses on socially optimal policies accounting for avoidance activities aside from the standard extensive margin.

systematic analysis of the determinants of enforcement, apprehension and crime rates. I show that there is a unique equilibrium, affording unambiguous comparative statics. Since I do not focus on the details of an offense, my framework provides a stepping stone, laying out the basic ingredients, to study specific and important economic and social problems, such as drug dealing, corruption and malfeasance. While I do not pursue normative analysis, the model — specially the supply and demand framework — can be used for transparent normative analysis in the spirit of standard price-theoretic models.

I use the framework to derive testable implications, and examine the deterrent and marginal deterrent effect of diverse policies aiming to discourage illegal behavior. For example, a uniform increase in penalties reduces the number of law breakers, but may incentivize law breakers to commit more severe offenses. Less attractive outside options (e.g. caused by a higher unemployment rate, or by a better criminal alternative) increases the number of people breaking the law, who in turn shift to less serious offenses. Enforcement and crime move in opposite directions as enforcement cost or compensation rises.

Next, I also showed how different enforcement tactics (e.g. announcing crackdowns), or centralization of enforcement yield unambiguously worse outcomes (e.g. higher crime rates and lower enforcement levels) relative to decentralization when law enforcers are incentivized to apprehend law breakers. Finally, I explored the effects of positive spillovers in law enforcement on levels of apprehension and crime, finding that they effectively reduce law breaking.

This paper is the first to look at equilibrium outcomes in a decentralized structure of law enforcement. As such, it considers a simplified model to capture the conflict between law breakers and enforcers, and provides the groundwork for future research. For instance, I assumed risk-neutral enforcers who are rewarded every time they arrest a law breaker. While this compensation structure is intuitive, it might not be necessarily optimal. Addressing how decentralized enforcement is affected by different compensation schemes seems a fruitful direction for future research.[31] Equally well, depending on the application in mind, one could also extend the framework to examine how realistic frictions, such as moral hazard in law enforcement, or search frictions, affect individuals' behavior as well as market outcomes. Finally, I assume homogeneous law enforcers, and so in equilibrium all behave identically. Extending the framework allowing for heterogeneity would shed light on issues related to corruption and quality of enforcement (Becker and Stigler, 1974), as some enforcers are corrupted and others are not. All these avenues are left for future research.

[31] See Di Porto et al. (2013) for providing one of the first analyses towards this directions.

A Omitted Proofs

A.1 Proof of Lemma 1

Totally differentiating the detection chance in e yields: $d\wp/de = \wp_e + \wp_\sigma d\Sigma^*/de$. Next, differentiate (3) in σ to get: $\Pi_{\sigma\sigma} = r'' - \wp_{\sigma\sigma} f - 2\wp_\sigma f' - \wp f''$. We'll show that $\wp_{\sigma\sigma f} + \wp f'' \geq 0$, and so $\Pi_{\sigma\sigma} < 0$, since $r'' < 0 < f''$. First, $\mathcal{E}_\sigma(f) \geq 1 \geq \mathcal{E}_\sigma(\wp)$, since the respective functions f and $\wp$ are increasing-convex and increasing-concave that start form the origin ($f(0) = 0 = \wp(e, 0)$), and thus their secants (f/σ and $\wp/\sigma$) rise and fall in σ. Next,

$$\wp_{\sigma\sigma} f + \wp f'' \geq 0 \iff \frac{f''}{f} \geq -\frac{\wp_{\sigma\sigma}}{\wp} \iff \frac{f''}{f'} \cdot \frac{f'}{f} \geq -\frac{\wp_{\sigma\sigma}}{\wp_\sigma} \cdot \frac{\wp_\sigma}{\wp} \tag{16}$$

The last inequality holds, for $\mathcal{E}_\sigma(f') \geq 1 \geq \mathcal{E}_\sigma(\wp_\sigma)$, and $\mathcal{E}_\sigma(f) \geq 1 \geq \mathcal{E}_\sigma(\wp)$. So $\Pi_{\sigma\sigma} < 0$.

Now, since $\Pi_{\sigma\sigma} < 0$, the Implicit Function Theorem yields $d\Sigma^*/de = -\Pi_{\sigma e}/\Pi_{\sigma\sigma}$, where $\Pi_{\sigma e} = -\wp_{\sigma e} f - \wp_e f'$. It is not hard to see that

$$\frac{d\wp}{de} \geq 0 \iff \frac{d\Sigma^*}{de} \geq -\frac{\wp_e}{\wp_\sigma} \iff -\wp_e r'' + \wp_e \wp_{\sigma\sigma} f + \wp_e \wp_\sigma f' + \wp_e \wp f'' \geq \wp_\sigma \wp_{\sigma e} f$$

Since $\wp$ is log-submodular (i.e., $\wp_{e\sigma} \leq \wp_\sigma \wp_e/\wp$), it is enough to show that

$$-\wp_e r'' + \wp_e \wp_{\sigma\sigma} f + \wp_e \wp_\sigma f' + \wp_e \wp f'' \geq \frac{\wp_\sigma^2 \wp_e}{\wp} f \iff -\wp_e r'' + \wp_e \wp_{\sigma\sigma} f + \wp_e \wp f'' + \wp_e \wp_\sigma f' - \frac{\wp_\sigma^2 \wp_e}{\wp} f \geq 0$$

The first term is positive since $r'' < 0$; the sum of second and third terms is positive, by (16); and the sum of fourth and fifth terms is positive, since $\mathcal{E}_\sigma(f) \geq 1 \geq \mathcal{E}_\sigma(\wp)$. □

A.2 Proof of Lemma 3

STEP 1: THE SUPPLY OF CRIME. First, the supply $\mathcal{K}^S(\cdot)$ in (5) is continuous, for $\bar{\omega}(\cdot)$ is continuous by the Envelope Theorem, and $G(\cdot)$ is an atomless distribution. Moreover, as $e \to \infty$, the marginal criminal $\bar{\omega}(e) \to \bar{\omega}_L > 0$ (for $\wp(e, \sigma) \to 1$), where $\omega_L \equiv \max_\sigma r(\sigma) - f(\sigma)$, and so the supply $\mathcal{K}^S(e) \to G(\bar{\omega}_L) > 0$. Also, supply $\mathcal{K}^S(e) \uparrow \infty$ as $e \downarrow 0$, for $\wp(e, \sigma) \downarrow 0$.

STEP 2: THE DEMAND FOR CRIME. By the Implicit Function Theorem, the FOC (8) yields a smooth best reply map $(\sigma, \kappa) \mapsto \mathcal{E}$. For a fixed $\sigma > 0$, $\mathcal{E}(\kappa, \sigma) \downarrow 0$ as $\kappa \downarrow 0$, since $c(0) = 0$. Also, enforcement level explodes $\mathcal{E}(\kappa, \sigma) \to \infty$ as $\kappa \to \infty$, for $V_e(e, \sigma, \kappa) \to \infty$. Now invert the best reply $\mathcal{E}(\cdot)$ to get $\mathcal{K}^D(\cdot)$. By the Inverse Function Theorem, $\mathcal{K}^D(\cdot, \sigma)$ is continuous and differentiable, and obeys: $\mathcal{K}^D(e, \sigma) \downarrow 0$ as $e \downarrow 0$, and $\mathcal{K}^D(e, \sigma) \to \infty$ as $e \to \infty$.

STEP 3: SINGLE CROSSING. Define the *excess of supply function* $\Delta\mathcal{K}(e|\sigma) \equiv \mathcal{K}^S(e) -$

$\mathcal{K}^D(e,\sigma)$. By Step 1–2, $\Delta\mathcal{K}(e|\sigma) \downarrow -\infty$ as $e \to 0$; and $\Delta\mathcal{K}(e|\sigma) \uparrow \infty$ as $e \uparrow \infty$. So by the Intermediate Value theorem, there exists $e^* \in (0,\infty)$ with $\Delta\mathcal{K}(e^*|\sigma) = 0$. Finally, e^* is unique, since $\Delta\mathcal{K}$ is monotone decreasing in effort e (for $\mathcal{K}_e^S < 0 < \mathcal{K}_e^D$). □

A.3 Proof of Proposition 3(a) Finished

Claim A.1 *Suppose enforcement efficacy rises. Then the Σ^* locus shifts down more than $\mathcal{C}^*$ does, and so the offense severity σ^* falls.*

Proof: Fix σ and log-differentiate $\mathcal{K}^S(e|\varphi) \equiv \mathcal{K}^D(e,\sigma|\varphi)$ in φ, and differentiate the Σ^* locus (3) in φ:

$$\frac{de}{d\varphi}\bigg|_{\mathcal{C}^*}\left(\frac{c''}{c'} - \frac{\wp_{ee}}{\wp_e} - \frac{g}{G}\bar{\omega}'\right) = \frac{g}{G}\bar{\omega}_\varphi + \frac{\wp_{e\varphi}}{\wp_e} \quad \text{and} \quad \frac{de}{d\varphi}\bigg|_{\Sigma^*}\left(\frac{\wp_{\sigma e}}{\wp_e} + \frac{f'}{f}\right) = -\frac{\wp_\varphi}{\wp_e}\left(\frac{\wp_{\sigma\varphi}}{\wp_\varphi} + \frac{f'}{f}\right) \tag{17}$$

Now, $\wp_{\sigma e}/\wp_e \leq \wp_\sigma/\wp = \wp_{\sigma\varphi}/\wp_\varphi$, since $\wp$ is log-modular in (σ,φ). Thus, $de/d\varphi|_{\Sigma^*} \leq -\wp_\varphi/\wp_e$ by (17). Next, since $\wp_{e\varphi}/\wp < (g/G)\wp_\varphi f$ and $c''/c' - \wp_{ee}/\wp_e > 0$, I have that $de/d\varphi|_{\mathcal{C}^*} > -(g/G)\bar{\omega}_\varphi/[(g/G)\bar{\omega}'] = -\bar{\omega}_\varphi/\bar{\omega}_\varphi = -\wp_\varphi/\wp_e$, given (17). Altogether, $de/d\varphi|_{\mathcal{C}^*} > de/d\varphi|_{\Sigma^*}$, and so the equilibrium severity σ^* falls. □

Claim A.2 *Suppose enforcement efficacy rises. The Σ^* shifts left more than $\mathcal{C}^*$, and thus the equilibrium enforcement e^* falls.*

Proof: Fix e and log-differentiate $\mathcal{K}^S(e|\varphi) \equiv \mathcal{K}^D(e,\sigma|\varphi)$ in φ, and differentiate (3) in φ to get:

$$-\frac{\wp_{e\sigma}}{\wp_e}\cdot\frac{d\sigma}{d\varphi}\bigg|_{\mathcal{C}^*} = -\frac{g}{G}\wp_\varphi f + \frac{\wp_{e\varphi}}{\wp_e} \qquad \text{and} \qquad \frac{d\sigma}{d\varphi}\bigg|_{\Sigma^*} = -\frac{\Pi_{\sigma\varphi}}{\Pi_{\sigma\sigma}} \tag{18}$$

The enforcement falls if the Σ^* locus shift more than the $\mathcal{C}^*$ locus, or $d\sigma/d\varphi|_{\Sigma^*} < d\sigma/d\varphi|_{\mathcal{C}^*}$:

$$\frac{d\sigma}{d\varphi}\bigg|_{\mathcal{C}^*} < \frac{d\sigma}{d\varphi}\bigg|_{\Sigma^*} \iff \wp_{e\sigma}\left(\frac{\Pi_{\sigma\varphi}}{\Pi_{\sigma\sigma}} - \frac{\wp_{e\varphi}}{\wp_{e\sigma}}\right) > -\frac{g}{G}\wp_\varphi f$$

We'll show that the left hand side of the right expression above is non-negative. First, since the detection chance $\wp$ is log-submodular in (e,σ), log-supermodular in (e,φ), and $\wp_{e\sigma} \geq 0$:

$$\wp_\sigma\Pi_{\sigma\varphi} \geq \wp_\varphi\Pi_{\sigma\sigma} \quad \Longrightarrow \quad \frac{\Pi_{\sigma\varphi}}{\Pi_{\sigma\sigma}} - \frac{\wp_{e\varphi}}{\wp_{e\sigma}} \geq 0$$

For since $\Pi_{\sigma\varphi} < 0$ and $\Pi_{\sigma\sigma} < 0$, I have $\wp_{e\sigma}\Pi_{\sigma\varphi} \geq (\wp_e\wp_\sigma/\wp)\Pi_{\sigma\varphi} \geq (\wp_e\wp_\varphi/\wp)\Pi_{\sigma\sigma} \geq \wp_{e\varphi}\Pi_{\sigma\sigma}$. Next, since $\Pi_{\sigma\varphi} = -(\wp_{\sigma\varphi}f + \wp_\varphi f) < 0$, $\Pi_{\sigma\sigma} = r'' - \wp_{\sigma\sigma}f - 2\wp_\sigma f' - \wp f'' < 0$, and $\wp$ is log-modular in (σ,φ) (i.e., $\wp_{\sigma\varphi}/\wp_\varphi = \wp_\sigma/\wp$), I have $\wp_\sigma \Pi_{\sigma\varphi} \geq \wp_\varphi \Pi_{\sigma\sigma}$ since

$$-\wp_\sigma f(\wp_\sigma/\wp + f'/f) \geq r'' - \wp_{\sigma\sigma} f - 2\wp_\sigma f' - \wp f'' \iff r'' - \wp_{\sigma\sigma} f + \wp_\sigma f(\wp_\sigma/\wp - f'/f) - \wp f'' \leq 0$$

which holds since $r'', \wp_{\sigma\sigma} \leq 0 < f''$ and $\wp_\sigma/\wp < f'/f$. □

Claim A.3 *Suppose that $\wp$ is log-modular in (e,σ). If enforcement efficacy rises (i.e., φ rises), then the crime rate κ falls.*

Proof: Let $\Delta\mathcal{K} \equiv \mathcal{K}^S - \mathcal{K}^D$ denotes the *excess of supply*. Totally differentiating the optimal severity locus Σ^* and the market clearing locus $\mathcal{C}^*$ yields, and using Cramer's rule:

$$\begin{pmatrix} \Pi_{\sigma e} & \Pi_{\sigma\sigma} \\ \Delta\mathcal{K}_e & \Delta\mathcal{K}_\sigma \end{pmatrix} \begin{pmatrix} e_\varphi \\ \sigma_\varphi \end{pmatrix} = \begin{pmatrix} -\Pi_{\sigma\varphi} \\ -\Delta\mathcal{K}_\varphi \end{pmatrix} \Longrightarrow \begin{pmatrix} e_\varphi \\ \sigma_\varphi \end{pmatrix} = det^{-1} \begin{pmatrix} \Delta\mathcal{K}_\sigma & -\Pi_{\sigma\sigma} \\ -\Delta\mathcal{K}_e & \Pi_{\sigma e} \end{pmatrix} \begin{pmatrix} -\Pi_{\sigma\varphi} \\ -\Delta\mathcal{K}_\varphi \end{pmatrix}$$

where $det^{-1} \equiv \Pi_{\sigma e}\Delta\mathcal{K}_\sigma - \Pi_{\sigma\sigma}\Delta\mathcal{K}_e < 0$, since $\Delta\mathcal{K}_\sigma = -\mathcal{K}^D_\sigma > 0$, by (9); and $\Delta\mathcal{K}_e = \mathcal{K}^S_e - \mathcal{K}^D_e < 0$, by (6) and (9). Next, since in equilibrium $\kappa = \mathcal{K}^S$, the crime rate falls iff $d\mathcal{K}^S/d\varphi = g(\bar{\omega}_e e_\varphi + \bar{\omega}_\varphi) < 0$. Using my expression for e_φ, and doing some algebra:

$$\bar{\omega}_e e_\varphi + \bar{\omega}_\varphi < 0 \iff \Delta\mathcal{K}_\sigma(\Pi_{\sigma e}\bar{\omega}_\varphi - \Pi_{\sigma\varphi}\bar{\omega}_e) + \Pi_{\sigma\sigma}(\Delta\mathcal{K}_\varphi\bar{\omega}_e - \Delta\mathcal{K}_e\bar{\omega}_\varphi) > 0$$

Now, observe that $\mathcal{K}^S_\varphi\bar{\omega}_e = \mathcal{K}^S_e\bar{\omega}_\varphi$, given (4) and (5). Thus, $\Delta\mathcal{K}_\varphi\bar{\omega}_e - \Delta\mathcal{K}_e\bar{\omega}_\varphi = \mathcal{K}^D_e\bar{\omega}_\varphi - \mathcal{K}^D_\varphi\bar{\omega}_e < 0$, since $\mathcal{K}^D_e > 0 > \mathcal{K}^D_\varphi, \bar{\omega}_\varphi, \bar{\omega}_e$, given (4) and (9). Therefore, it is enough to show that $\Pi_{\sigma e}\bar{\omega}_\varphi - \Pi_{\sigma\varphi}\bar{\omega}_e \geq 0$, since $\Delta\mathcal{K}_\sigma > 0$.

$$\Pi_{\sigma e}\bar{\omega}_\varphi - \Pi_{\sigma\varphi}\bar{\omega}_e = gf[\wp_\varphi(\wp_{\sigma e} f + \wp_e f') - \wp_e(\wp_{\sigma\varphi} f + \wp_\varphi f')] = gf^2[\wp_\varphi\wp_{\sigma e} - \wp_e\wp_{\sigma\varphi}] = 0$$

The last equality is true, since $\wp$ is log-modular in (σ, e) and (σ,φ). □

A.4 Proof of Proposition 3(b) Finished

Claim A.4 *If criminal competence rises, then the enforcement falls in the short-run.*

Proof: Fix e and log-differentiate $\mathcal{K}^S(e|\varphi) \equiv \mathcal{K}^D(e,\sigma|\varphi)$ in φ, and differentiate (3) in φ to get:

$$-\frac{\wp_{e\sigma}}{\wp_e} \cdot \frac{d\sigma}{d\varphi}\bigg|_{\mathcal{C}^*} = -\frac{g}{G}\wp_\varphi f + \frac{\wp_{e\varphi}}{\wp_e} = \frac{\wp_{e\varphi}}{\wp_e} \quad \text{and} \quad \frac{d\sigma}{d\varphi}\bigg|_{\Sigma^*} = -\frac{\Pi_{\sigma\varphi}}{\Pi_{\sigma\sigma}}$$

The enforcement falls if the $\mathcal{C}^*$ locus shifts right more than Σ^*, or $d\sigma/d\varphi|_{\Sigma^*} < d\sigma/d\varphi|_{\mathcal{C}^*}$:

$$\frac{d\sigma}{d\varphi}\bigg|_{\mathcal{C}^*} > \frac{d\sigma}{d\varphi}\bigg|_{\Sigma^*} \iff \Pi_{\sigma\varphi}\frac{\wp_{e\sigma}}{\wp_e} - \Pi_{\sigma\sigma}\frac{\wp_{e\varphi}}{\wp_e} < 0$$

Since the detection chance $\wp$ is log-submodular in (e, φ), log-submodular in (σ, e) it is enough to show that $\Pi_{\sigma\varphi}\wp_\sigma - \Pi_{\sigma\sigma}\wp_\varphi < 0$, given the right side of the expression above. Since $\Pi_{\sigma\varphi} =$ and $\Pi_{\sigma\sigma} =$:

$$\Pi_{\sigma\varphi}\wp_\sigma - \Pi_{\sigma\sigma}\wp_\varphi = (-\wp_{\sigma\varphi}\wp_\sigma f + \wp_\sigma\wp_\varphi f') + (\wp_\varphi\wp_{\sigma\sigma} f + \wp_\varphi\wp_\sigma f'') - \wp_\varphi r'' < 0$$

Since $r'' < 0$, it is enough to show that each parenthesized term is negative. Consider the first parenthesized term:

$$(-\wp_{\sigma\varphi}\wp_\sigma f + \wp_\sigma\wp_\varphi f') = \wp_\sigma\wp_\varphi f\left(-\frac{\wp_{\sigma\varphi}}{\wp_\varphi} + \frac{f'}{f}\right) < \wp_\sigma\wp_\varphi f\left(-\frac{\wp_\sigma}{\wp} + \frac{f'}{f}\right) < 0$$

The inequality follows since $\wp$ is log-submodular in (σ, φ), and $\mathcal{E}_\sigma(f) \geq 1 \geq \mathcal{E}_\sigma(\wp)$, and $\wp_\varphi < 0$. Now consider the second parenthesized term:

$$(\wp_\varphi\wp_{\sigma\sigma} f + \wp_\varphi\wp_\sigma f'') = \wp_\varphi\wp_\sigma f\left(\frac{\wp_{\sigma\sigma}}{\wp_\sigma} + \frac{f''}{f}\right) = \wp_\varphi\wp_\sigma f\left(\frac{\wp_{\sigma\sigma}}{\wp_\sigma} + \frac{f''}{f'}\frac{f'}{f}\right) < 0$$

The inequality holds since $\mathcal{E}_\sigma(f'), \mathcal{E}_\sigma(f) \geq 1 > |\mathcal{E}_\sigma(\wp_\sigma)|$, and $\wp_\varphi < 0$. □

A.5 Proof of Proposition 4 Finished

Proposition 8 (Greater Enforcement Payoff) *If the payoff per criminal apprehended B rises, then the enforcement level rises, while the offense severity and crime rate fall. The arrest rate rises if g/G is small enough, but it falls if g/G is sufficiently high.*

Proof: Suppose that B rises. In the (κ, e)-space, the demand $\mathcal{K}(e,\sigma)^D = c'(e)/[\wp_e(e,\sigma)B]$ shifts left, while the supply $\mathcal{K}^S(e)$ is unaffected by B. Thus, for a fixed offense severity σ, the market clearing enforcement rises. So in the (σ, e)-space, this translates into a left shift of the market clearing locus $\mathcal{C}^*$ (right panel of Figure 5). Since the severity locus Σ^* is unaffected, by (3), the enforcement level e^* rises whereas the offense severity σ^* falls.

Next, the crime falls since by (6), $d\mathcal{K}^S(e)/dB = g\bar{\omega}' de/B < 0$. Finally, since the detection chance $\wp$ falls as e falls along Σ^* (Lemma 1), the detection chance rises as B rises. In the short-run, where κ is fixed, the arrest rate rises. However, in the long-run, since κ falls and $\wp$ rises, the net effect on the arrest rate is ambiguous. By the same logic of the proof of Proposition 4, the arrest rate rises when $g/G \sim 0$, but it falls when $g/G \sim \infty$ — for intuitively, an increase in the enforcement level induces excessive compliance (see (7)). □

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