

## A Theory of Crime and Vigilance<sup>†</sup>

By JORGE VÁSQUEZ\*

*This paper develops a theory of crime in which potential victims elect their vigilance levels. When vigilance expenses are greater than expected property losses, an increase in penalties raises crime, namely, a criminal Laffer curve emerges. This curve is higher and peaks earlier when victims face higher costs. Thus, the government may wish to subsidize vigilance rather than increase penalties. Indeed, an increase in penalties may shift the vigilance levels further away from their socially optimal ones. Finally, the crime rate first rises and then falls in the property value at stake, which is consistent with the empirical evidence. (JEL D91, H76, K42)*

Previous research on the economics of crime has emphasized the role of penalties and policing as key drivers of criminal behavior; however, private self-protection by individuals—or *vigilance*—is also an important determinant that has received significantly less attention in the literature. Indeed, recent evidence indicates that vigilance expenditures against crime are sizable and even greater than public expenditures. Over and above property losses, Anderson (2011) estimates that \$480 billion is spent annually on vigilance efforts in the United States, in addition to public expenditures.<sup>1</sup> Also, as we move toward a more digitized and globally connected economy, cybercrime has become increasingly costly, and it presents a pressing issue for corporations, which spend major resources on security (Makridis and Dean 2018).<sup>2</sup>

Understanding vigilance efforts is essential for improving policy interventions. Consider, for example, an increase in legal penalties or policing. On one hand, the intended effect of this legal policy is clear: to reduce criminal activity via greater deterrence (Becker 1968). On the other hand, this policy blunts individuals'

\*Smith College (email: [jvasquez@smith.edu](mailto:jvasquez@smith.edu)). Leslie Marx was coeditor for this article. I am indebted to Lones Smith for his guidance and support. I thank five anonymous referees for valuable comments and suggestions that helped improve the paper. I also thank Gonzalo Cisternas, Steven Durlauf, Louis Kaplow, Pino Lopomo, Antonio Penta, Nicola Persico, Daniel Quint, Ben Vollard, Marek Weretka, Mariyana Zapryanova, and all seminar participants at Bank of Canada, Collegio Carlo Alberto, ESEM 2017 (Lisbon), Indiana University-Kelley, NBER Summer Institute, PUC-Chile, Quebec Political Economy Conference (5 edition), Smith College, the University of Wisconsin-Madison, UC-Lovain, University of Chile-CEA, and Virginia Tech for their insightful comments. This paper builds on the first chapter of my Ph.D. thesis, which was joint work with my advisor, Lones Smith, and was previously circulated as a manuscript entitled "Crime and Vigilance." Any errors are my own.

<sup>†</sup>Go to <https://doi.org/10.1257/mic.20190339> to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

<sup>1</sup>Altogether, Anderson (2011) estimates the cost of crime to be around \$1 trillion in the United States.

<sup>2</sup>According to estimates of the Center for Strategic and International Studies, cybercrime costs the global economy up to \$600 billion a year. This represents a \$155 billion increase since 2014 (McAfee 2018).

incentives to be vigilant and thereby raises the expected gains of a criminal offense. Altogether, the final effect is ambiguous, for there is a *vigilance force* that will inevitably undermine the efficacy of this policy, potentially crowding out the government's goal. Thus, to correctly evaluate the effect of any policy aimed at alleviating crime, it is fundamental to know (1) how both market and technological forces influence potential victims' incentives to self-protect, (2) how those incentives impact behavior and welfare, and (3) the conditions under which policies lead to undesirable outcomes.

This paper develops a novel model of crime centered on the competition between potential criminals and potential victims. By showing precisely how the different competing forces interact, this model provides a new vehicle for analyzing policy and its impact on crime. In this model, criminals choose whether to attempt a crime, while potential victims decide how vigilant to be. While formally specified as a game, the model admits a natural *supply and demand* interpretation that yields a rich and tractable microfounded framework for performing equilibrium, welfare, and policy analyses.

Randomness is a crucial aspect of crime. First, few of us are crime victims in any period. According to the Bureau of Justice Statistics, in 2017, only 10.8 percent of 123 million households experienced one or more property victimizations in the United States. The stochastic nature of crime is captured by assuming random encounters of criminals and victims. These encounters give rise to the *attempted crime rate*, which influences potential victims' level of vigilance. Second, randomness plays another key role because not all attempted crimes succeed (e.g., one often fails to break into a house or steal a car).<sup>3</sup> The *failure rate* of an attempted crime determines the expected gains from crime and is positively affected by potential victims' vigilance.

As in Becker (1968), potential criminals have an extensive margin: whether to engage in crime. Stricter punishments or increased policing make crime less appealing and so formally raise the expected legal costs of crime. I assume that the incentives to attempt a crime vary across potential criminals by considering heterogeneous *opportunity costs*. On the other side, potential victims elect vigilance levels to secure, perhaps imperfectly, their property. Given vigilance, the efficacy to frustrate an attempted crime is heterogeneous: potential victims vary by their self-protection *technologies*. Finally, because criminals are often surprised by the vigilance they encounter, I assume that vigilance efforts are unobserved by criminals. This yields a rich and tractable benchmark model through which one can unveil economic forces likely to be present even in instances where vigilance is observed, as shown in Section VIII.

All told, I explore the equilibrium of a model in which a continuum of heterogeneous potential criminals each chooses whether to engage in crime, while a continuum of heterogeneous potential victims each elects their level of vigilance. The number of criminals then fixes the attempted crime rate, while vigilance determines the failure rate. Potential victims are hurt by a greater attempted crime rate, while

<sup>3</sup>A recent test verified that TSA security prevents only 5 percent of criminal attempts; see [www.vox.com/2016/5/17/11687014/tsa-against-airport-security](http://www.vox.com/2016/5/17/11687014/tsa-against-airport-security).

greater vigilance impedes criminal gains. In *equilibrium*, all agents optimize, and the crime rate reflects the number of successful attempted crimes. This model naturally applies to property crime, where property can be defined in a broad sense ranging from physical goods (e.g., cars) to digital ones (e.g., data).

I reinterpret the equilibrium as a competitive equilibrium, depicted in a metaphorical, yet microfounded, supply and demand framework. Due to the natural asymmetry of the market, the average failure rate plays the role of a price for potential criminals, while the attempted crime rate plays the same role for potential victims. Optimal criminal behavior induces a *downward-sloping supply of crime*, as fewer crimes are committed when failing is more likely. Similarly, optimal vigilant behavior induces an *upward-sloping demand for vigilance* because a greater victimization chance elicits a more vigilant response. The supply-demand crossing yields a unique *market equilibrium*, rendering unambiguous all analyzed comparative statics. Contrary to standard supply and demand analyses, here “trade” is not win-win; rather, criminal offenses constitute an economic good for criminals but a bad for potential victims. This distinction is fundamental when performing welfare and policy analyses.

*Overview of the Results.*—The model delivers new testable predictions. To see this, consider the well-known claim that aggregate and individual criminal activity should fall when legal penalties or policing increase or, more generally, when the expected punishment rises (Becker 1968). By contrast, in this paper a higher expected punishment deters criminal entry, shifting the supply curve down, while leaving the demand curve unaffected. So greater expected punishment discourages vigilance,<sup>4</sup> as documented by Vollaard and Koning (2009), raising the marginal profitability of every offense. On balance, there are fewer criminals, but each is less likely to fail to compensate for the greater punishment. When the demand curve is sufficiently elastic, the effect on the crime rate is nonmonotone. Precisely, Proposition 1 shows that the crime rate is a *hump-shaped* function of the expected punishment, resembling a *criminal Laffer curve*. Thus, an increase in punishment can actually lead to more crime.<sup>5</sup>

Next, Proposition 2 provides a simple sufficient condition to determine *when* the criminal Laffer curve is upward sloping: an increase in expected punishment raises the crime rate if total vigilance expenses are no less than realized property losses. Proposition 3 then studies the micro-effects of raising the expected punishment. While the attempted crime rate is common to all potential victims, the ultimate property loss faced by them depends, largely, on their own vigilant behavior. I find that the *Lafferian force* is stronger for potential victims with superior technologies who will face more crime after an increase in punishment.

<sup>4</sup>Becker (1968, 188) argued that improved law enforcement has an ambiguous effect on total offenses. He conjectured that more policing would be partially offset by a drop in private vigilance. Pushing this logic further, this paper shows when such vigilance displacement leads to undesirable outcomes. See Nagin (2013) and Chalfin and McCrary (2017) for excellent surveys of policing and crime.

<sup>5</sup>“Lafferian effects” have been found in other contexts. For instance, in the tax compliance literature (Graetz, Reinganum, and Wilde 1986; Andreoni, Erard, and Feinstein 1998), an increase in taxes can lead to an increase in the tax compliance rate. In the industrial organization literature, it has been argued that consumer protection policies (e.g., price caps) can actually harm consumers (Armstrong, Vickers, and Zhou 2009).

Because the endogenous vigilance efforts by potential victims lie at the heart of this paper, I explore how vigilance costs, or technologies, impact equilibrium outcomes. First, unlike punishment, when vigilance costs rise, or individuals with inferior technologies are more abundant, the demand falls but the supply is unaffected. Thus, there are more criminals, each with a lower chance of failing, such that the crime rate rises. As a result, the criminal Laffer curve shifts up and peaks earlier (Proposition 4).

I then explore a variation in both the material losses for victims and gains for criminals, or in the *stakes of crime*. This naturally applies, for example, to a fixed class of goods, as the value of the goods varies for owners and criminals alike. Likewise, this change helps inform policies that improve the recovery rate of stolen goods, as they clearly lower the stakes of crime. In general, an increase in the stakes of crime impacts both sides of the market: facing a greater loss, potential victims grow more vigilant, while criminals respond to the increased gains with more offenses. The demand and supply curves increase, and so does the failure rate. However, Proposition 5 shows that both the crime rate and the attempted crime rate are nonmonotone and highest for mid property values. These findings may help to explain, e.g., why most stolen cars are typically in the midrange for prices<sup>6</sup> and at a more aggregated level, why as a country's wealth rises crime initially rises and then, eventually, falls. This nonmonotone pattern has been observed in many countries, in particular, in the United States from 1960 until now (Farrell, Tilley, and Tseloni 2014).<sup>7</sup> From a policy perspective, improving the recovery rate of stolen goods (i.e., lowering the stakes) seems to be effective only for low-value property.

When vigilance is unobservable, potential victims benefit from one another's vigilance efforts, as everyone faces the same attempted crime rate independent of their choices. These positive externalities lead to an underprovision of total vigilance (Clotfelter 1978; Shavell 1991). Nonetheless, it is important to tie this insight to policy: if legal punishment were to rise, should we expect more or less underprovision of vigilance? Proposition 6 examines how the optimal failure rate and the degree of underprovision are affected by the expected punishment. Interestingly, raising punishment can shift the equilibrium levels of vigilance further away from their optimal ones. By contrast, policies targeted to directly aid the demand side, such as subsidizing or mandating vigilance (Vollaard and van Ours 2011), can reduce or even eliminate the underprovision gap.

Some forms of vigilance are noticeable, such as installing a fence or an outside camera on one's property. When vigilance is observed, criminals can effectively target their offenses. But, because criminals are ultimately affected by the success rate of their attempts—which depends upon both the chosen vigilance and the victim's *private* cost-technology—observing low vigilance is not necessarily good news for criminals. In equilibrium, potential criminals use the information contained in vigilance choices to make failure rate inferences and target their offenses, whereas

<sup>6</sup>According to the National Insurance Crime Bureau, from 2006 to 2017 the Honda Civic and Accord have consistently been the most stolen vehicles in the United States; see [www.nicb.org/news/reports-statistics](http://www.nicb.org/news/reports-statistics).

<sup>7</sup>Among many competing hypotheses, these authors conclude that vigilance (security, in their paper) is the most likely explanation for why crime has declined in many countries.

potential victims understand this when choosing their optimal vigilance intensity. This novel *signaling* force is embedded into the model, which yields a tractable framework to perform equilibrium analysis. Proposition 7 gives an equilibrium characterization, uncovering new forces and qualitative differences between observable and unobservable vigilance. By means of example, the levels of crime and vigilance are lower and higher, respectively, when vigilance is observable. Also, observed vigilance leads to overprovision of vigilance, which can be ameliorated with harsher punishment. Finally, since the observed vigilance case also admits a supply and demand interpretation, the crowding out of vigilance owed to an increase in punishment is prone to emerge, by common logic.

*Outline.*—Section II sets up the model, and Section III performs equilibrium analysis. Section IV characterizes the criminal Laffer curve and explores its determinants. Section V examines the effects of vigilance costs and property stakes. Section VI performs welfare analysis, whereas Section VII studies the underprovision of vigilance and the role of policy. Finally, Section VIII discusses the observable vigilance case. Section IX concludes and discusses directions for empirical and theoretical work. All omitted proofs and analyses are in the Appendix.

## I. Literature Review

The groundbreaking paper by Becker (1968) gave the first economic analysis of crime. Accounting for the optimal criminal response to changes in punishment and the probability of capture, he explored the socially optimal level of law enforcement expenditures. Becker's focus is on the supply side: in his analysis, the levels of crime are determined by the criminal optimization. The demand side is missing in his analysis. Ehrlich (1981) later posited a reduced-form downward demand for crime, where the net payoff per offense acts as the price, and argued that the number of offenses is, indeed, an equilibrium variable.<sup>8</sup> This point was echoed and further developed by Cook (1986). Yet the optimization of potential victims is absent in those papers.<sup>9</sup> In my analysis, the induced supply and demand curves explicitly arise from both the criminals' and victims' optimization problems, respectively. In addition, in my microfounded framework, I exploit the randomness of crime by modeling both probabilistic encounters and stochastic vigilance efficacy. This allows me to originally disentangle crimes and attempted crimes, which is crucial if the goal is to unveil forces that shape policy interventions. In particular, the aforementioned criminal Laffer curve emerges precisely because the crime rate and attempted crime rate are not forced to co-move.

<sup>8</sup>Conceptually, our demand curves are different: In Ehrlich (1981, 309), "the demand schedule for offenses represents the average potential payoff per offense at alternative frequencies of offenses"; see Ehrlich (2010) for a survey. This story suggests that victims' vigilance directly reduces criminal material payoffs. By contrast, here the key effect of vigilance, like burglar systems, is to raise the failure chance of an attempted crime and so reduce the *expected* criminal gain. See Clotfelter (1977) for early work on the topic.

<sup>9</sup>Hotte, Valognes, and van Ypersele (2003) develop a market for offenses. While their supply follows from criminal entry, their demand reflects the criminal optimization. So their market equilibrium takes vigilance as given.

There is a small literature that examines how equilibrium vigilance levels are affected by whether vigilance is observed by criminals and how these levels compare to benchmarks. In a model with homogeneous victims and a fixed pool of homogeneous criminals, Shavell (1991) finds that, relative to the levels that minimize the victims' losses, vigilance is overprovided when it is observable by criminals and underprovided when unobservable. More recently, building on Lacroix and Narceau (1995), Baumann and Friehe (2013) find that vigilance may be underprovided even if it is observable when there is asymmetric information between victims and criminals. Relatedly, this literature also focuses on how the observability of vigilance may divert crime to less protected victims (Koo and Png 1994; Hotte and Van Ypersele 2008).<sup>10</sup> My analysis differs from these papers along three dimensions. First, their focus is on the demand: they consider a *fixed* number of criminals and so a constant attempted crime rate. In my analysis, the attempted crime rate is an endogenous variable that responds to vigilance and is key in explaining crime rates. Second, this literature considers an exogenous theft chance function, whereas in my paper it is shaped by a criminal participation constraint. Finally, these papers do not pursue welfare or policy analysis.

Following the normative approach to crime (see Polinsky and Shavell 2000 for a survey), there is a small literature that focuses on optimal law enforcement and rational victim behavior (Ben-Shahar and Harel 1995; Hylton 1996; Garoupa 2003; Helsley and Strange 2005). There, the government is another player who may choose the level of policing and penalties in order to optimize a welfare function. Instead, my analysis considers an exogenous government and explores how the jointly determined levels of crime and vigilance respond to different intensities of policing and penalties. This allows me to provide a full characterization of how and when vigilance will crowd out the government's policy objective.

Finally, somewhat related, Knowles, Persico, and Todd (2001) focus on the interaction between police and heterogeneous potential criminals and seek to identify whether police stops exhibit racial or statistical discrimination. Their criminals decide whether to carry contraband, and the probability of a police search influences their choices. The models differ in most respects but share the feature that the criminal extensive margin and the police search decision (like vigilance) are jointly determined. Persico (2002) gives an equilibrium analysis of police fairness and crime, while O'Flaherty and Sethi (2008) develop an equilibrium model to explain racial disparities in robbery, in which victims can choose to comply or resist. Burdett, Lagos, and Wright (2003) develop an equilibrium theory that jointly determines wages, unemployment, and crime, assuming exogenous private vigilance efforts. Quercioli and Smith (2015) develop a counterfeiting model, focusing on a money passing game, assuming homogeneous "bad" and "good" guys. They study how the counterfeiting rate depends on the notes' denomination.

<sup>10</sup>For empirical studies, see Ayres and Levitt (1998); Vollaard and van Ours (2011); Cook and MacDonald (2011); Gonzalez-Navarro (2013); Zimmerman (2014); van Ours and Vollaard (2015); Priks (2015); Borker (2018); Domínguez (2020). See Draca and Machin (2015) for a recent survey.



## II. The Model

**Players and Matching:** Consider an economy in which the ownership of a single good is dispersed among a large population of risk-neutral agents. I call those endowed with the good *potential victims* and those not so endowed *potential criminals*. The value of the property at stake is  $M > 0$ , where  $M$  represents a *property loss* for potential victims and a *criminal gain* for potential criminals. Thus, the focus is naturally on property crime, such as theft, burglary, cybercrime, and not on so-called “victimless” crimes such as speeding, contraband, and drug offenses. I consider a static environment, with the usual understanding that it can be seen as the steady state of a dynamic model.

Following Becker (1968), potential criminals simply have an extensive margin—they choose whether to attempt a crime. If so, they shall be labeled as *criminals*. Potential criminals are heterogeneous and differ by their outside option, or opportunity cost  $c \geq 0$ . This cost subsumes, for example, the expected forgone income from the legal sector. The *cost mass distribution*  $F$  has a density  $f(c) \equiv F'(c) > 0$  on  $[0, \infty)$ . Since the mass of criminals is endogenous,  $F$  need not be a probability distribution with unit mass. As in Becker (1968), criminals are apprehended with *capture probability*  $p \in [0, 1]$  and, if so, face a *penalty*  $x \geq 0$ , where the latter is the monetary equivalent of the punishment. Altogether, criminals treat  $c + \ell$  as the fixed cost of attempting a crime, where  $\ell \equiv px \geq 0$  is the expected legal *punishment* and it is naturally bounded by the criminal gain  $M$ , namely,  $\ell \leq M$ .

Randomness plays two roles in this model. First, criminals randomly meet potential victims as part of a decentralized pairwise random matching model. All potential victims are criminal targets facing the same endogenous *attempted crime rate*  $\alpha \geq 0$ , namely, the expected number of attempted crimes per capita. Second, if a potential victim faces an attempted crime, she can randomly deter it by, for example, placing bars on windows, installing home alarms, or by avoidance and mental alertness.<sup>11</sup> All such actions determine a level of *vigilance*. Because the model is stationary, vigilance is a flow cost, or the annuity value of initial one-shot costs, and so it is a direct subtraction from utility.

Any attempted crime against a potential victim can either fail or succeed in stealing the victim’s property. A potential victim is able to impact the failure rate of an attempted crime by being vigilant. Potential victims vary by their private cost-technology, indexed by  $\xi \in [0, 1]$ . Types  $\xi$  have a probability distribution  $G$  and density  $g(\xi) \equiv G'(\xi) > 0$  on  $(0, 1)$ . A potential victim with technology  $\xi$  can stop an attempted crime with probability  $\phi \in [0, 1]$  if her vigilance equals  $V(\phi|\xi) \geq 0$ .<sup>12</sup> That is,  $V(\phi|\xi)$  is the *vigilance cost function* of securing an *individual failure rate*  $\phi$  for type  $\xi$ . I assume a monotone and strictly convex cost  $V_\phi(\phi|\xi)$ ,  $V_{\phi\phi}(\phi|\xi) > 0$  for  $\phi > 0$ , with  $V(0|\xi) = V_\phi(0|\xi) = 0$  and  $V_\phi(1|\xi) < \infty$  for every  $\xi$ . While marginal vigilance costs rise, I posit that its elasticity  $(\phi V_{\phi\phi}/V_\phi)$  falls in  $\phi$ . This assumption will ensure that potential victims are more sensitive to crime when the attempted crime rate is higher. More technically, it implies that

<sup>11</sup> Each adult spends 1 minute and 50 seconds locking and unlocking doors each day (Anderson 2011).

<sup>12</sup> E.g.,  $\xi$  may index an invariant layer of private security embedded in the property good.

marginal costs are log-concave,  $[\log(V_\phi)]_{\phi\phi} \leq 0$ , and so cannot increase too rapidly; also, it forces a limit  $\lim_{\phi \downarrow 0} V_{\phi\phi}/V_\phi >^{\phi} 1$ .

I further assume that potential victims with higher types  $\xi$  face greater costs and a faster increase in marginal costs:  $V_\xi(\phi|\xi) > 0$  and  $V_{\phi\phi\xi}(\phi|\xi) > 0$  for  $\phi > 0$ . The latter condition implies that higher types  $\xi$  have greater marginal costs  $V_{\phi\xi}(\phi|\xi) > 0$  for  $\phi > 0$ .<sup>13</sup> Finally, I assume that the marginal cost elasticity  $(\phi V_{\phi\phi}/V_\phi)$  increases in  $\xi$ . All told, vigilance cost  $V(\phi|\xi)$  rises in  $\xi$ , and its marginal  $V_\phi$  is supermodular in  $(\phi, \xi)$  and log-supermodular in  $(\phi, \xi)$ .<sup>14</sup> These assumptions not only simplify the analysis but will discipline the comparative statics. Any convex geometric function  $V(\phi|\xi) = (1 + \xi)\phi^\gamma, \gamma \geq 1$ , meets all these properties.

Criminals cannot observe potential victims' types  $\xi$  and vigilance level,<sup>15</sup> and so each criminal conjectures that a type  $\xi$  potential victim elects vigilance to ensure an individual failure rate  $\phi(\xi)$ . Since  $\xi$  varies across potential victims, criminals expect that their attempted crimes will fail with chance  $\varphi \equiv \int_0^1 \phi(\xi) dG(\xi) \in [0, 1]$ , the average or expected failure rate. Naturally, the criminal success rate is  $1 - \varphi$ , and the success odds are  $(1 - \varphi)/\varphi$ . The actual crime rate  $\kappa \equiv \alpha(1 - \varphi) \in [0, 1]$  denotes the mass of successful attempted crimes.

**Optimizations and Equilibrium:** Because losses realize when a crime succeeds,<sup>16</sup> a potential victim chooses the failure rate  $\phi \in [0, 1]$  to minimize the expected property losses of crime  $\alpha(1 - \phi)M$  plus vigilance cost  $V(\phi|\xi)$ :<sup>17</sup>

$$(1) \quad \mathcal{L}(\phi, \alpha, \xi) \equiv \alpha(1 - \phi)M + V(\phi|\xi).$$

Reflecting an underlying competitive assumption that no potential victim impacts the attempted crime rate  $\alpha$ , potential victims choose  $\phi$  to minimize (1), taking  $\alpha$  as given. In this respect, vigilance (or private defense) is different from law enforcement (or public defense), which surely has aggregate effects; see, e.g., Polinsky and Shavell (2000). To isolate the impact of vigilance on crime, policing behavior is unmodeled and simply embedded into the legal punishment  $\ell$  with the understanding that policing is positively associated with  $\ell$ .<sup>18</sup>

<sup>13</sup>Since  $V_\phi(0|\xi) = 0$  for all  $\xi$ , it follows that  $V_{\phi\xi}(\phi|\xi) = \int_0^\phi V_{\phi\phi\xi}(t|\xi) dt > 0$  for  $\phi > 0$ .

<sup>14</sup>A real valued function  $x \mapsto h$  on a lattice  $X \subseteq \mathbb{R}^n$  is supermodular (submodular) if  $h(\max\{x, x'\}) + h(\min\{x, x'\}) \geq (\leq) h(x) + h(x')$ . When  $h$  is twice differentiable, then  $h$  is supermodular (submodular) if  $h_{x_i x_j}(x) \geq (\leq) 0$  for all  $i \neq j$ , by Topkis (1998). These definitions are strict if the inequalities are strict. A positive function  $h > 0$  is log-supermodular (log-submodular) if  $\log(h)$  is supermodular (submodular).

<sup>15</sup>In reality, vigilance also subsumes observable actions. For instance, installing a fence or bars on windows are observable to criminals. Yet these observable actions are likely to be similar across potential victims who have similar characteristics (e.g., in a residential neighborhood). Thus, given a "submarket," the encounters between criminals and victims can be seen as random, and the variation of actual crimes across victims can be seen as a result of unobservable and idiosyncratic vigilance actions (e.g., installing deadbolt locks on house doors or following "best avoidance practices"). Section VIII explores the observable vigilance case.

<sup>16</sup>In practice, victims could still bear losses (e.g., psychological costs) even if an attempted crime fails. This consideration can be incorporated into the model without sacrificing any insight. In this sense, the model analyzes the case in which losses from failed crimes are small relative to losses from successful ones.

<sup>17</sup>With risk-averse potential victims, the problem is slightly different: a victim chooses  $\phi$  to maximize expected utility  $\alpha(1 - \phi)u(w - M) + \alpha\phi u(w) - V(\phi|\xi)$ , for some increasing and concave utility function  $u(\cdot)$ , and income  $w > 0$ . Relaxing the model in this dimension is not essential for this paper's results.

<sup>18</sup>While law enforcement may also respond to changes in the attempted crime rate (e.g., hire police officers or adjust police patrol patterns), most individuals can vary their vigilance significantly quicker. Thus, at least from a



A potential criminal with cost  $c$  cannot observe the failure rates produced by potential victims, and so he takes the average failure rate  $\varphi$  as given and attempts a crime if and only if his expected *net criminal profits*  $\Pi(\varphi|c) \geq 0$ , where

$$(2) \quad \Pi(\varphi|c) \equiv (1 - \varphi)M - \ell - c.$$

The *marginal potential criminal*  $\bar{c}(\varphi) \equiv \max\{(1 - \varphi)M - \ell, 0\}$  earns nonpositive criminal profits. Thus, naturally, potential criminals attempt a crime provided their cost  $c \leq \bar{c}$ .<sup>19</sup>

Notice the embedded causation. The potential victims' losses reflect the aggregated criminal behavior, via the attempted crime rate, and the criminal profits are only impacted by the average failure rate. Also, the only spillovers of potential victims onto each other are indirectly channeled by the impact on criminal behavior, and conversely, criminals only affect other criminals indirectly via the potential victims' vigilance. Likewise, law enforcement indirectly impacts potential victims to the extent that it affects criminal participation.

Altogether, I examine equilibria of the simultaneous move interaction between potential victims and criminals. Potential victims independently and simultaneously choose their failure rates, anticipating the attempted crime rate, while potential criminals independently and simultaneously choose whether to attempt crime, taking as given the average failure rate. Formally, an *equilibrium* is a pair  $(\phi^*(\cdot), \bar{c}^*)$  such that (i) for each potential victim  $\xi$ , the individual failure rate  $\phi^*(\xi)$  minimizes losses  $\mathcal{L}(\phi, \alpha^*, \xi)$  in (1), given the attempted crime rate  $\alpha^* = F(\bar{c}^*)$ , and (ii) a potential criminal attempts a crime if and only if his cost is  $c \leq \bar{c}^*$ , where profits  $\Pi(\varphi^*|\bar{c}^*) = 0$  in (2), for an average failure rate  $\varphi^* = \int_0^1 \phi^*(\xi) dG(\xi)$ .

### III. A Supply and Demand Equilibrium Analysis

**The Demand Curve:** Consider the potential victims' optimization problem (1). Given an attempted crime rate  $\alpha \geq 0$ , the best response for a potential victim  $\xi$  is an *optimal failure rate*  $\hat{\phi}(\alpha|\xi) \in [0, 1]$  that minimizes losses  $\mathcal{L}(\phi, \alpha|\xi)$ . Clearly, if there is no attempted crime, then individuals elect no vigilance, or  $\hat{\phi}(0|\xi) = 0$ . If the optimal failure rate is interior, i.e.,  $\hat{\phi}(\alpha|\xi) \in (0, 1)$ , then it obeys the first-order condition for (1):

$$(3) \quad \mathcal{L}_\phi(\hat{\phi}, \alpha|\xi) = -\alpha M + V_\phi(\hat{\phi}|\xi) = 0.$$

The second-order condition holds by strict convexity of  $V$ . Notice that the marginal losses from crime are negative when the attempted crime rate  $\alpha > V_\phi(1|\xi)/M$ , and so the optimal failure rate is interior when  $0 < \alpha < V_\phi(1|\xi)/M$ , and obeys (3). Since  $V_\phi(0|\xi) = 0$ , the optimal failure rate  $\hat{\phi}(\alpha|\xi) = 0$  if  $\alpha = 0$ , and

short-run perspective, punishment  $\ell$  can be seen as exogenous for decision-making.

<sup>19</sup>With risk-averse criminals, profits turn to  $(1 - \varphi)[p\pi(M - x) + (1 - p)\pi(M)] + \varphi[p\pi(-x) + (1 - p)\pi(0)]$ , for some increasing and concave  $\pi(\cdot)$ , capture chance  $p$ , and penalty  $x$ . Notice that with or without risk-aversion, an increase in  $\varphi$  lowers profits, making the focus on risk-neutrality an innocuous simplification.

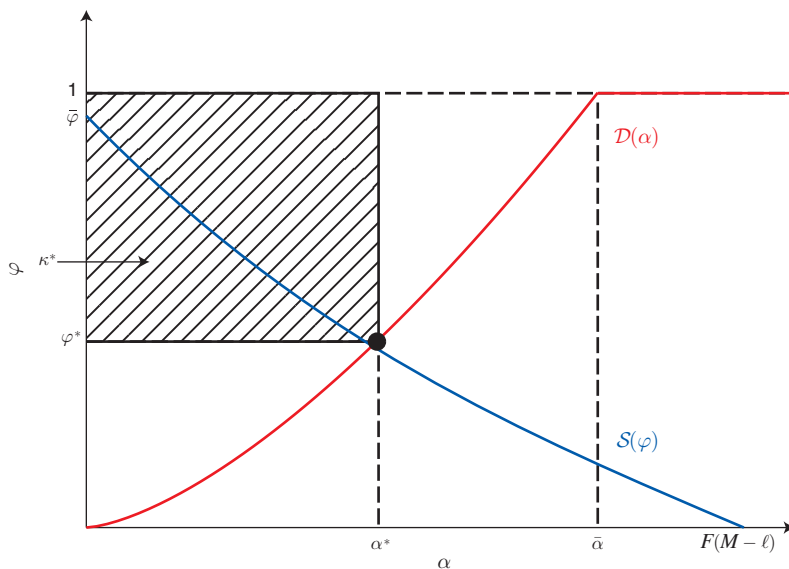


FIGURE 1. EQUILIBRIUM EXISTENCE AND UNIQUENESS

Notes: The demand slopes upward, while the supply slopes down. The crime rate  $\kappa^* = (1 - \varphi^*)\alpha^*$  is the dashed area of successful attempted crimes.

$\hat{\phi}(\alpha|\xi) > 0$  if  $\alpha > 0$ . Also,  $\hat{\phi}(\alpha|\xi)$  rises from zero as  $\alpha$  rises from zero since  $\mathcal{L}_{\alpha\phi} = -M < 0$ . Now, potential victims with higher types  $\xi$  have higher vigilance cost  $V(\cdot|\xi)$  and marginal cost  $V_\phi(\cdot|\xi)$  and thus choose a lower failure rate, i.e.,  $\hat{\phi}_\xi(\alpha|\xi) < 0$ , and face greater losses from crime (by the Envelope theorem applied to (1)). Finally, the individual level of vigilance expenditures  $V(\hat{\phi}(\alpha|\xi)|\xi)$  is hump-shaped in  $\xi$ , provided the attempted crime rate  $\alpha$  is neither too high nor too low (Claim A.1).

The optimal failure rate  $\hat{\phi}(\alpha|\xi)$  can naturally be interpreted as an induced *individual demand* for vigilance. At the aggregate, the market *demand* for vigilance, given the attempted crime rate  $\alpha$ , is captured by

$$(4) \quad \mathcal{D}(\alpha) \equiv \int_0^1 \hat{\phi}(\alpha|\xi) dG(\xi).$$

The demand vanishes with no attempted crime,  $\mathcal{D}(0) = 0$ , and is continuous and upward sloping (see Figure 1) because crime is an economic bad. It hits perfect security  $\mathcal{D}(\alpha) = 1$  once all types  $\xi$  choose  $\hat{\phi}(\alpha|\xi) = 1$ , i.e., when  $\alpha \geq \bar{\alpha} \equiv V_\phi(1|1)/M$ . Thus, the threshold  $\bar{\alpha}$  is called the *attempted crime ceiling*. As potential victims take more precautions when they risk losing a more valuable good, the demand  $\mathcal{D}$  increases in the property loss  $M$ .

When is the market demand  $\mathcal{D}$  more or less elastic? Consider two type-distributions for potential victims, namely,  $G_L$  and  $G_H$  with respective densities  $g_L$  and  $g_H$ . Appendix A.A2 shows that *demand*  $\mathcal{D}$  is less elastic with  $G_H$  than  $G_L$  if  $g_H(\xi)/g_L(\xi)$  is increasing. Recall that when the likelihood ratio  $g_H(\xi)/g_L(\xi)$  is monotone, then  $G_H$  dominates  $G_L$  in the sense of first-order and second-order stochastic

dominance (see, e.g., Athey 2002). Thus, the market demand falls when potential victims with higher vigilance costs are more abundant because the individual demand  $\hat{\phi}(\alpha|\xi)$  is decreasing in  $\xi$ . That is, the demand is lower with  $G_H$  than  $G_L$ . Likewise, by standard results, the demand falls when vigilance costs are less dispersed in the population, provided the individual demand  $\hat{\phi}(\alpha|\xi)$  is convex in type  $\xi$  (this is, e.g., the case when vigilance costs  $V(\phi|\xi)$  are geometric in  $\phi$  for all  $\xi$ ).

**The Supply Curve:** Given an expected failure rate  $\varphi$ , the marginal potential criminal is  $\bar{c} = (1 - \varphi)M - \ell$  if and only if  $\varphi \leq \bar{\varphi} \equiv 1 - \ell/M$ , the *failure rate ceiling*. No crimes are attempted when the expected failure rate is too high,  $\varphi > \bar{\varphi}$ . Hence, the *supply* of crime is

$$(5) \quad \mathcal{S}(\varphi) \equiv F((1 - \varphi)M - \ell),$$

for  $\varphi \leq \bar{\varphi}$ , and  $\mathcal{S}(\varphi) \equiv 0$  for  $\varphi > \bar{\varphi}$ . The supply curve simply reflects the (almost everywhere) differentiable and increasing map from the marginal criminal  $\bar{c}$  to the attempted crime rate  $\alpha = F(\bar{c})$ . As noted after (3), if the failure rate  $\varphi$  is less than  $\bar{\varphi}$ , the attempted crime rate is at most  $V_\phi(1|1)/M$  and thus is boundedly finite, as seen in Figure 1.

The supply curve is downward sloping, falling in  $\varphi$  (see Figure 1) and vanishing as  $\varphi \uparrow \bar{\varphi}$ . It rises in the criminal gain  $M$  but falls in punishment  $\ell$ . Also, *the supply elasticity falls when the cost distribution  $F$  improves in the monotone likelihood ratio sense* (Appendix A.A3). Therefore, the supply is lower when more potential criminals have higher opportunity costs.

Notice that, for potential victims, the failure rate  $\varphi$  plays the role of a demand quantity, while the attempted crime rate  $\alpha$  serves as a price. But conversely, for potential criminals,  $\varphi$  acts as a supply price, while  $\alpha$  acts as a quantity. Thus, while a more elastic supply in Figure 1 is more horizontal per usual, a more elastic demand is more vertical.

The falling supply and rising demand curves imply that *there exists a unique equilibrium*  $(\phi^*(\cdot), \bar{c}^*)$ , balancing market forces (see Appendix A.A4). Figure 1 depicts this unique equilibrium. In such equilibrium, the crime rate  $\kappa^*$  is the rectangular shaded area  $(1 - \varphi^*)\alpha^*$ .

**Example 1:** Consider a criminal-cost distribution  $F(c) = c$ , a quadratic vigilance cost  $V(\phi|\xi) = (1 + \xi)\phi^2/2$ , and a potential-victim-type distribution  $G(\xi) = (1 + \xi)^2/4$ . To obtain simple closed-form solutions, I assume property value  $M \leq 2$  so that, in the unique equilibrium, almost all potential victims elect  $\hat{\phi}(\alpha^*|\xi) < 1$  for all punishment  $\ell \in [0, M]$ .

Under this parameterization, the optimal failure rate that solves (3) is given by  $\hat{\phi}(\alpha|\xi) = \alpha M / (1 + \xi)$ . This expression is well defined for all types  $\xi$  as long as  $\alpha \leq 1/M$ , a condition that is verified in equilibrium. Now, integrating across types  $\xi$  yields the market demand curve  $\varphi = \mathcal{D}(\alpha) = 3\alpha M / 4$ , displayed in (4). The supply curve (5) easily follows  $\alpha = \mathcal{S}(\varphi) = (1 - \varphi)M - \ell$ . Thus, intersecting the demand and supply curves determines the unique equilibrium:  $\alpha^* = 4(M - \ell) / (4 + 3M^2)$  and  $\varphi^* = 3(M - \ell)M / (4 + 3M^2)$ .

Finally, I verify that  $\alpha^* \leq 1/M$ . Since punishment  $\ell \in [0, M]$  and  $M \leq 2$ , it follows that  $\alpha^* M = 4(M - \ell)M / (4 + 3M^2) \leq 4M^2 / (4 + 3M^2) \leq 1$ .

**Remark 1 (Vigilance and Policing):** This paper assumes that potential victims' vigilance has no impact on probability that a criminal is captured by the police. That is, the probability of capture  $p$  does not depend on the failure rate  $\varphi$ . One could imagine cases in which vigilance and policing have complementary effects. For instance, an alarm could turn on during a burglary, making burglars more likely to be apprehended. This would directly affect the supply curve. Appendix D considers this possibility and shows that the key property of the supply curve (namely, that it is downward sloping) does not depend on whether vigilance and policing are complements. Thus, the qualitative insights that this paper provides remain valid, and assuming independent policing and vigilance is a harmless simplification. However, clearly the levels of crime and vigilance will change, as criminals will be more likely to get caught whenever they fail.

#### IV. The Criminal Laffer Curve

Understanding the deterrence effects of different policies aimed at alleviating crime is a challenge that lies at the heart of the economics of crime literature (Becker 1968). Policies not only differ in their effects on crime but also in their implementation costs. For instance, it is arguably the case that deterring individuals from criminal participation is more economical than incapacitating them in prisons. Chalfin and McCrary (2017) argue that “offenders who are deterred from committing crime in the first place do not have to be identified, captured, prosecuted, sentenced, or incarcerated ...” (p. 5). While there are several ways of making crime less attractive, perhaps the most direct one is increasing either legal penalties ( $x$ ) or policing ( $p$ ) and thereby the expected legal punishment of crime ( $\ell$ ).

So motivated, in this section, I study the equilibrium effects of raising an offense's expected punishment,  $\ell = px$ . I explore how potential victims' vigilance impacts the determination of crime rates and the efficacy of public policy, as the expected punishment rises.

It is well-known in the economics of crime literature that, at least theoretically, an increase in the expected punishment must unambiguously reduce crime (Becker 1968). This insight owes to the decision theory nature of most crime models and also to the uniform treatment of attempted and actual crimes. Indeed, when attempted and actual crimes are carefully disentangled,<sup>20</sup> an increase in punishment unambiguously lowers only the attempted crime rate  $\alpha$ . Meanwhile, potential victims respond to variations in  $\alpha$ , reducing their vigilance as the attempted crime rate  $\alpha$  falls. As a result, *the crime rate  $\kappa$  need not be monotone*. So legal policies that appear effective ex ante may actually be ineffective ex post.

<sup>20</sup>See, e.g., Farrell (2016) and Tseloni et al. (2017) for empirical research that accounts for this difference.

Proposition 1 characterizes how increases in expected punishment impact individual behavior and market outcomes. Before stating the result, say that the demand  $\mathcal{D}$  is *sufficiently elastic* if its elasticity is greater than the criminal success odds, namely,  $\mathcal{E}_\alpha(\mathcal{D}) \geq (1 - \mathcal{D})/\mathcal{D}$ .

**PROPOSITION 1:** *If expected legal punishment  $\ell$  rises, then the failure rate  $\varphi^*$  and the attempted crime rate  $\alpha^*$  fall. The crime rate  $\kappa^*$  is a hump-shaped function of legal punishment  $\ell \in [0, M]$  if the demand  $\mathcal{D}$  is sufficiently elastic for low enough punishments; otherwise, the crime rate is monotone decreasing in  $\ell$ .*

For an intuition, consider two extreme cases. In the “police state,” massive legal penalties hold the attempted crime rate  $\alpha^*$  very low, and so decreases in legal penalties lead to proportionately large increases in  $\alpha^*$ , and thus the crime  $\kappa^* = (1 - \varphi^*)\alpha^*$  rises. In other words, if potential victims are overprotected by police, their behavior is not responsive enough to variations in penalties. At the opposite extreme, in the “Wild West,” policing is very slight, and crime is held at bay by individual vigilance—the quick draw. In this case, the criminal success rate  $1 - \varphi^*$  is extremely low, and so decreases in legal penalties lead to proportionately large increases  $1 - \varphi^*$ , and thus the crime rate  $\kappa^* = (1 - \varphi^*)\alpha^*$  falls. In less extreme cases, the impact of increasing criminal costs on the crime rate  $\kappa^* = (1 - \varphi^*)\alpha^*$  is unclear, a priori. Proposition 1 clarifies this, showing that, as seen in Figure 2, the equilibrium crime rate is a hump-shaped function of the expected punishment, resembling a *criminal Laffer curve*.<sup>21,22</sup>

The criminal Laffer curve systematically characterizes the trade-offs between crime rates and expected punishment and rationalizes the empirical finding that better policing displaces private vigilance (e.g., Vollaard and Koning 2009). It shows that the efficacy of public policy depends critically on the current level of policing and penalties for a criminal offense.<sup>23</sup> It also highlights the interaction between two opposing market forces that govern the competitive nature of crime. While raising expected penalties has a positive deterrent effect ex ante, since fewer crimes are attempted, increased penalties also has a negative deterrent effect ex post—as more attempted crimes succeed. The relative magnitude of these positive and negative effects on any given crime is, thus, an empirical question. Notice that when the expected punishment is below a critical value, the negative effect is dominant but becomes weaker as the punishment rises. The strength of these effects depends on the demand elasticity. The next result provides a simple *sufficient* condition that determines *when* the negative effect dominates, or the criminal Laffer curve is upward sloping in punishment  $\ell$ . Define the *total vigilance expenses* as  $\int_0^1 V(\hat{\phi}(\alpha|\xi)|\xi) dG(\xi)$ .

<sup>21</sup> See Ayres (2016) for a related and intuitive discussion of the criminal Laffer curve.

<sup>22</sup> Because  $\ell \equiv px$ , the crime rate  $\kappa^*$  will also be hump shaped in  $p$ , as  $\ell$  is proportional to  $p$ . That is, a criminal Laffer curve would arise between the crime rate  $\kappa^*$  and the probability of capture  $p$ .

<sup>23</sup> For instance, as seen in the right panel of Figure 2, the crime rate  $\kappa^*$  is low but strictly positive in the “Wild West,” whereas the crime rate is zero when punishment  $\ell$  exceeds property gain  $M$ , as attempting a crime would be suboptimal regardless of victims’ vigilance. Thus, if the goal is to minimize the crime rate, then raising police protection and penalties, so that  $\ell \geq M$ , would achieve this objective.

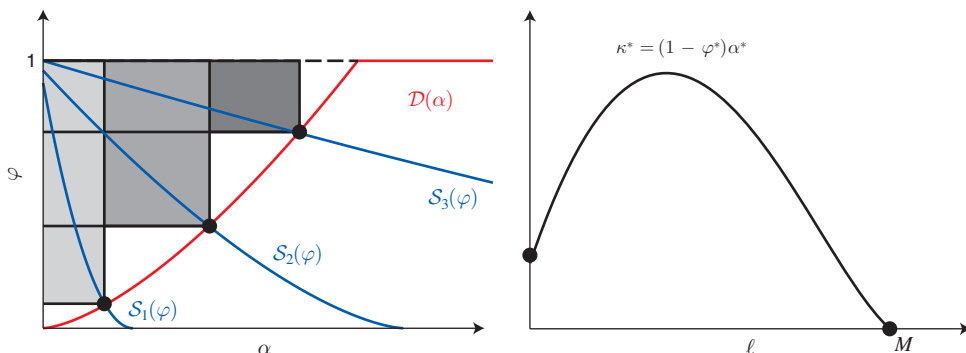


FIGURE 2. PUNISHMENT AND THE CRIMINAL LAFFER CURVE

Notes: Left: When expected punishment  $\ell$  rises from  $\ell = 0$ , supply shifts left from  $S_3(\varphi)$ , and the failure rate  $\varphi^*$  and attempted crime rate  $\alpha^*$  fall. Right: The crime rate  $\kappa^*$  is hump shaped and so highest for mid punishment levels.

**PROPOSITION 2:** *The crime rate rises when the expected legal punishment rises, if the equilibrium property losses are less than total vigilance expenses.*

Proposition 2 provides a condition, based on observable variables, that sheds light on when increasing expected punishment will be more than crowded out by a reduction in potential victims' vigilance. This result builds on Proposition 1. Indeed, raising penalties leads to higher crime rates when the demand curve is sufficiently elastic (Proposition 1). Thus, Proposition 2 essentially identifies when this is the case, by leveraging the structure of the model and equilibrium conditions.<sup>24</sup> In addition, as a corollary, Proposition 2 implies a lower bound on the average efficacy of vigilance. To see this, notice that the average effect of vigilance is to reduce expected property losses from  $\alpha M$  to  $\alpha(1 - \varphi)M$ , and so naturally a cost-benefit analysis implies that total vigilance expenses must be less than the average property value preserved by potential victims,  $\alpha\varphi M$ .<sup>25</sup> Consequently, if property losses  $\alpha(1 - \varphi)M$  are less than total vigilance expenses, then the average failure rate  $\varphi$  must be greater than 0.5. In other words, a criminal attempt must fail with more than 50 percent chance.

Proposition 2 is of practical interest and raises awareness regarding the use of harsher punishment. For example, according to Ayres and Levitt (1998), in the United States the property loss of vehicles with LoJack was roughly \$1,000. They estimate a theft rate in LoJack cities of 0.025 and a \$97 annuity equivalent of the \$600 LoJack installation fee. This exceeds the expected property loss of \$25. So

<sup>24</sup> In the same spirit, Persico (2002) shows how to use the distributions of income opportunities to criminals to identify the relative elasticity of crime to policing in two groups of people.

<sup>25</sup> Optimal behavior implies that vigilance spending is less than  $\alpha\varphi M$  but not necessarily less than property losses  $\alpha(1 - \varphi)M$ . For example, one could pay, say, \$50 for an alarm that *reduces* the expected loss in, say, \$100, although the total expected loss with the alarm is only, say, \$10. This decision is perfectly rational since not paying \$50 implies that the expected loss would increase from \$10 to \$110.



Proposition 2 predicts that, if penalties for auto theft had risen, the auto theft rate, if anything, would have increased.<sup>26</sup>

Finally, to close this section, I ask what happens at the individual level? Notice that the criminal Laffer curve aggregates the individual responses of potential victims to variations in expected punishment. While the attempted crime rate is common to all potential victims, the ultimate property loss faced by them depends on their own vigilant behavior. This means that potential victims effectively face different victimization rates. Thus, if punishment were to increase, which types of potential victims will end up facing more crime? The next result shows that, as punishment rises, the crime rate rises for potential victims with low vigilance costs and falls for those with high costs; furthermore, the proportion of potential victims facing less crime rises. To wit, the negative ex post effect of punishment dominates for potential victims with superior self-protection technologies.

**PROPOSITION 3:** *For any punishment  $\ell \in [0, m]$ , there exists a potential victim critical type  $\bar{\xi} \in [0, 1]$  such that a marginal increase in punishment  $\ell$  leads to higher crime rates for all potential victims  $\xi \leq \bar{\xi}$  and to lower crime rates for all potential victims  $\xi > \bar{\xi}$ . Moreover, the critical type  $\bar{\xi}$  is decreasing in punishment  $\ell$ .*

By Proposition 3, an increase in punishment could lead to more crime for some types of potential victims, even if the total crime rate decreases. Intuitively, potential victims with low vigilance costs (i.e., low  $\xi$ ) are more sensitive to variations in the attempted crime rate than those with high costs, leading them to thwart fewer attempted crimes.

**Example 2:** Consider the parameterization given in Example 1. The equilibrium crime rate  $\kappa^* = \alpha^*(1 - \varphi^*)$  is thus given by  $\kappa^* = 4(M - \ell)(4 + 3M\ell)/(4 + 3M^2)$ . The criminal Laffer curve is the map  $\ell \mapsto \kappa^*$ . Differentiating this curve with respect to punishment  $\ell$  yields  $\kappa_\ell^*(\ell) \equiv \partial\kappa^*/\partial\ell = -4(4 + 6M\ell - 3M^2)/(4 + 3M^2)^2$ . When property value  $M > \sqrt{4/3}$ , the criminal Laffer curve strictly rises from  $\ell = 0$  (i.e.,  $\kappa_\ell^*(0) > 0$ ). The reason is that, in such case, the demand curve  $\mathcal{D}(\alpha) = 3\alpha M/4$  is sufficiently elastic for  $\ell = 0$  (Proposition 1). Indeed, the demand elasticity  $\mathcal{E}_\alpha(\mathcal{D}) = 1$ , while the criminal success odds  $(1 - \mathcal{D})/\mathcal{D} = 4/(3M^2)$ ; therefore,  $\mathcal{E}_\alpha(\mathcal{D}) \geq (1 - \mathcal{D})/\mathcal{D}$  when property value  $M \geq \sqrt{4/3}$ .

<sup>26</sup>For another illustration, consider the overall motor vehicle theft in the United States. From the FBI, in 2017 the average property loss from motor vehicle theft was \$7,708; the theft rate was 237.4 per 100,000 inhabitants, or at least 470 per 100,000 car owners, i.e., a theft rate at least  $\kappa = 0.004$ . So the expected property losses are at least  $\kappa M = \$30$  (conservatively assuming stolen cars are a write-off). Thus, an increase in penalties would lead to more crime if average vigilance expenses exceed \$30. Since a car is on average replaced every 10 years, this amounts to a one-shot investment on car security greater than \$185.

## V. Comparative Statics

### A. The Role of Vigilance Costs

When potential victims face high vigilance costs, policymakers may wish to subsidize them or may seek to increase, for example, policing to counterbalance potential increases of crime. While more policing can, effectively, control the attempted crime rate, it is unclear whether the number of actual crimes would decrease due to the criminal Laffer curve. Self-protection technologies impact not only the vigilance levels but also their adjustment to changes in policy. These aspects should be given consideration when policy toward crime is considered.

When more potential victims face higher vigilance costs, the type distribution  $G$  naturally shifts. Consider two distributions for potential victims,  $G_L$  and  $G_H$ . Say that  $G$  rises if  $G$  shifts from  $G_L$  to  $G_H$  in the monotone likelihood ratio sense.

**PROPOSITION 4:** *When more potential victims face higher vigilance costs (i.e., when  $G$  rises): (i) the failure rate  $\varphi^*$  falls, while the crime rate  $\kappa^*$  and attempted crime rate  $\alpha^*$  both rise; and (ii) the criminal Laffer curve shifts up and left.*

As seen in the left panel of Figure 3, when  $G$  rises the failure rate  $\varphi^*$  falls, whereas the attempted crime rate  $\alpha^*$  and crime rate  $\kappa^*$  rise. The reason is that demand  $\mathcal{D}$  shifts down as the proportion of potential victims with high vigilance cost rises. Because supply  $\mathcal{S}$  is unaffected by  $G$ , the success of a criminal attempt rises, and so does the crime rate as more crimes are attempted and each succeeds with a higher probability. Consequently, the criminal Laffer curve shifts up (right panel of Figure 3), and it peaks sooner because  $G$  also affects the demand elasticity, as shown in Lemma A.2 in Appendix A.A8.

Thus, when more individuals have access to technologies that lower vigilance costs, the levels of crime fall. This prediction is consistent with the empirical finding that after the introduction of LoJack, the auto theft rate fell substantially in the United States (Ayres and Levitt 1998).<sup>27</sup> It is also in line with the observed decline in auto theft rates since 2003 in the United States, as newer cars have theft-immobilizer devices or part markings (Fujita and Maxfield 2012). Indeed, Farrell et al. (2011) find that the drop in auto theft over the last 20 years in the United States, Britain, and Australia is negatively correlated with an increase in anti-theft devices for cars. Likewise, van Ours and Vollaard (2015) find a large reduction of car theft in the Netherlands after the EU mandated that all new cars have electronic engine immobilizers.<sup>28</sup> In the same spirit, these authors also found in 2011 that burglary greatly fell in the Netherlands after a mandate for burglary-proof windows and doors in all home constructions.

<sup>27</sup>These authors find an elasticity of car theft with respect to LoJack adoption of  $-7$ . This sizable effect must reflect the elasticity of the supply curve, given Figure 3. Indeed, if the demand curve shifts left along the supply curve, the magnitude of the change in  $\alpha^*$  and  $\kappa^*$  is determined by the supply's slope.

<sup>28</sup>Accounting for theft diversion to older cars, van Ours and Vollaard find that the device lowered the overall rate of car theft on average by 40 percent during 1995–2008 in the Netherlands.

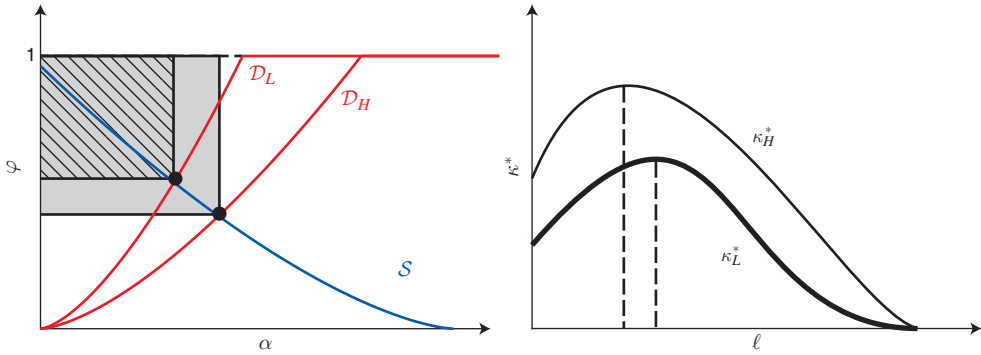


FIGURE 3. VIGILANCE COSTS AND THE CRIMINAL LAFFER CURVE

Notes: Left: When more victims face higher vigilance costs, i.e.,  $G$  rises from  $G_L$  to  $G_H$ , demand shifts right from  $\mathcal{D}_L$  to  $\mathcal{D}_H$  along  $S$ . The crime rate and attempted crime rate rise, while the failure rate falls. Right: The criminal Laffer curve shifts up and left.

In light of these results, policies aimed at reducing vigilance expenses are enticing, as they would not only decrease crime, but also they would reduce the region wherein an increase in penalties leads to more crime. In fact, demand-side interventions are particularly attractive compared to supply-side ones when vigilance spending exceeds property losses because, in such a case, an increase in punishment increases crime (Proposition 2), whereas a reduction in vigilance costs unambiguously lowers crime (Proposition 4).

### B. The Stakes of Crime

In this section, I explore the equilibrium effects of increasing the *stakes of crime*, or the property value  $M$ . For example, stealing a luxury vehicle would likely induce a greater loss to the owner and a greater gain to the criminal. The same logic should also hold for other offenses such as money theft and burglary. From a policy perspective, one would like to explore the effects of policies that, for example, improve the *recovery rate of stolen goods* in the sense of lowering both the expected victims' property losses and the criminals' gain.<sup>29</sup>

So motivated, say that *the stakes of crime rise* if  $M$  rises. In general, moving from low to high stakes affects both the demand and supply curves, complicating the comparative statics. The next result uncovers a criminal Laffer curve for the attempted crime rate.

**PROPOSITION 5:** *The failure rate  $\varphi^*$  rises as the stakes rise. If demand  $\mathcal{D}$  is sufficiently elastic for high enough  $M$ , then the attempted crime rate  $\alpha^*$  is a hump-shaped function of the stakes  $M$ ; the crime rate  $\kappa^*$  initially rises and then, eventually, falls*

<sup>29</sup> Assume that a stolen good can be recovered with chance  $r \in [0, 1]$ . The original model subsumes this by considering instead the effective loss  $(1 - r)M$  for victims and the effective gain  $(1 - r)M$  for criminals.

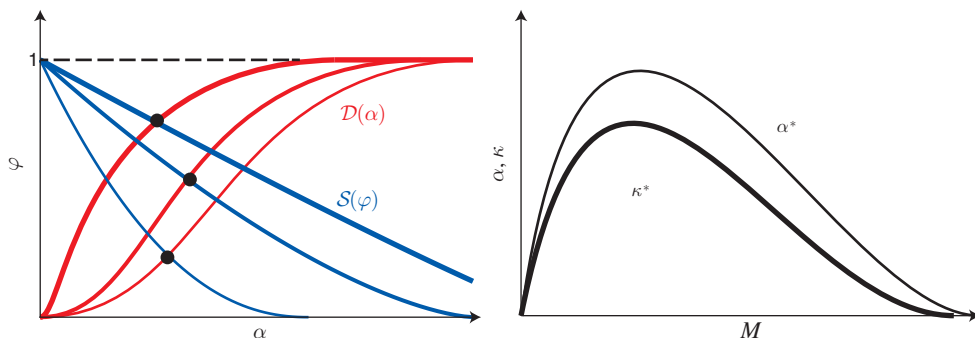


FIGURE 4. GREATER CRIMINAL GAINS AND PROPERTY LOSSES

Notes: For expositional clarity, the figure assumes punishment  $\ell = 0$ , and thus the failure rate ceiling  $\bar{\varphi} = 1$  for all  $M$ . Left: When the stakes of crime rise, demand  $\mathcal{D}$  shifts left, whereas supply  $\mathcal{S}$  shifts right (from thin to thick lines). Right: The failure rate  $\varphi^*$  rises, while the crime rate and attempted crime rate are nonmonotone.

as  $M$  rises. Potential victims with low cost  $\xi$  respond with more vigilance than high-cost ones, as stakes  $M$  rises.

First, as the stakes of crime rise, potential victims have more to lose, whereas potential criminals have more to gain. So, as seen in the left panel of Figure 4, potential victims raise their vigilance, shifting the demand curve  $\mathcal{D}$  up. Likewise, more potential criminals attempt a crime, moving the supply curve  $\mathcal{S}$  right. Thus, the failure rate  $\varphi^*$  unambiguously rises. The effect on the attempted crime rate is more subtle: *The attempted crime rate  $\alpha^*$  rises if and only if supply  $\mathcal{S}$  shifts up more than demand  $\mathcal{D}$  does.* Appendix A.A9 shows that supply shifts up more than demand does if and only if stakes are below a critical value. The attempted crime rate is, thus, hump shaped; see the right panel of Figure 4. The crime rate  $\kappa^*$  falls if stakes are high enough, for there are fewer attempts and each is more likely to fail. For low stakes, Appendix A.A9 shows that  $\kappa^*$  must be initially rising since supply  $\mathcal{S}$  vanishes as the stakes vanish.

Proposition 5 may explain, for example, why most stolen cars and attempted car thefts are consistently in the midrange (in price and year) in the United States, as reported by the National Insurance Crime Bureau.<sup>30</sup> Also, at a more aggregated level, this result suggests that, as a country's income rises (from low income) crime should initially rise and then, eventually, fall, *ceteris paribus*.<sup>31</sup> Finally, from a policy viewpoint, Proposition 5 argues that improving the recovery rate of stolen goods (i.e., lowering the stakes) is effective only for low-value property. Further connections with the empirical literature are discussed in Section IX.

<sup>30</sup> See Top Vehicles Stolen at <https://www.nicb.org/news/reports-statistics>.

<sup>31</sup> In many countries, the property crime rate trend has been nonmonotone from 1960 until now, as documented in, e.g., figures 2–4 in Farrell, Tilley, and Tseloni (2014). In particular, in the United States, crime peaked in the early 1990s when the average income per capita was around \$24,000. In the case of auto theft, the drop in crime afterward has been found to be negatively correlated with an increase in car security (Farrell et al. 2011).

**Example 3:** Consider the parameterization given in Example 1. I now focus on the effects of property values  $M$ . To this end, consider punishment  $\ell = 0$ . Then, the crime rate and attempted crime rate obey  $\kappa^* = 16M / (4 + 3M^2)^2$  and  $\alpha^* = 4M / (4 + 3M^2)$ , respectively. Differentiating  $\alpha^*$  in  $M$  yields  $\alpha_M^*(M) \equiv \partial\alpha^* / \partial M = 4(4 - 3M^2) / (4 + 3M^2)^2$ . Clearly,  $\alpha_M^*(M)$  falls in  $M$ , with  $\alpha_M^*(0) > 0 > \alpha_M^*(2)$ , implying that the attempted crime rate is hump shaped. As stated in Proposition 5, this result owes to the demand  $\mathcal{D}$  being sufficiently elastic for  $M \geq \sqrt{4/3}$ , as seen in Example 2. Finally, differentiating  $\kappa^*$  in  $M$  yields  $\kappa_M^*(M) \equiv \partial\kappa^* / \partial M = 4(16 - 36M^2) / (4 + 3M^2)^3$ . So  $\kappa_M^*(0) > 0 > \kappa_M^*(2)$ , namely, the crime rate initially rises and then, eventually, falls.

### VI. A Supply and Demand Welfare Analysis

Having examined the interplay between crime and vigilance, I now turn to its welfare consequences. The thrust of Becker (1968) was a treatise on the social costs of crime, focusing on the optimal public expenditures against crime. But, as argued in the introduction, vigilance expenses are an important source of social costs that have received much less attention among economists.

In this framework, the welfare loss of crime owes to criminal costs and vigilance costs.<sup>32</sup> Since the respective marginal criminal and attempted crime rate obey  $\bar{c} = (1 - \varphi)M - \ell$  and  $\alpha = F(\bar{c})$ , integrating by parts implies that total criminal costs are  $\int_0^{\bar{c}} (c + \ell)f(c) dc = \alpha(1 - \varphi)M - \int_0^{\bar{c}} F(c) dc$ . That is, criminal costs equal revenues  $\alpha(1 - \varphi)M$  minus profits  $\int_0^{\bar{c}} F(c) dc$ . Notice that criminal profits reflect criminal heterogeneity: a criminal  $c \in [0, \bar{c})$  makes profits  $\bar{c} - c$ , and thus total profits across all criminals amount to  $\bar{c}F(\bar{c}) - \int_0^{\bar{c}} cf(c) dc = \int_0^{\bar{c}} F(c) dc$ .

Total vigilance costs are  $\int_0^1 V(\hat{\phi}(\alpha|\xi) | \xi) dG(\xi)$ . Consequently, given the crime rate  $\kappa = \alpha(1 - \varphi)$ , the *social costs of crime* can be written as

$$(6) \quad \kappa M - \int_0^{\bar{c}} F(c) dc + \int_0^1 V(\hat{\phi}(\alpha|\xi) | \xi) dG(\xi).$$

Observe that the social costs of crime (6) can be reinterpreted as potential victims' expected losses (summing the left and right terms) less criminal profits (middle term).

Now let me suggestively depict the social costs in the supply and demand framework, developed in Section III. In any equilibrium (variables with a star), *criminal profits*  $\int_0^{\bar{c}^*} F(c) dc$  are captured by the area above equilibrium failure rate  $\varphi^*$  and below the value-scaled supply curve  $S(\varphi)M$  in Figure 5. That is,  $\int_0^{\bar{c}^*} F(c) dc = \int_{\varphi^*}^{\bar{\varphi}} S(\varphi)M d\varphi$ .<sup>33</sup> This crucially owes to the fact that crime is an economic bad for potential victims but an economic good for criminals.

Next, I depict vigilance expenses. For simplicity, consider an equilibrium in which all types  $\xi$  optimally choose imperfect vigilance, i.e.,  $\hat{\phi}(\alpha^*|\xi) < 1$ . Appendix A.A10 shows that *total vigilance expenses*  $v^*$  equal the area below  $\varphi^*$  and above

<sup>32</sup>There is no transfer loss of theft: the property loss to victims is balanced by equal gains to criminals.

<sup>33</sup>Indeed,  $\int_0^{\bar{c}^*} F(c) dc = \int_{\varphi^*}^{\bar{\varphi}} F(\bar{c}(\varphi)) \bar{c}_\varphi(\varphi) d\varphi$ , where  $F(\bar{c}(\varphi)) = S(\varphi)$  by (5) and  $\bar{c}_\varphi = -M$ .

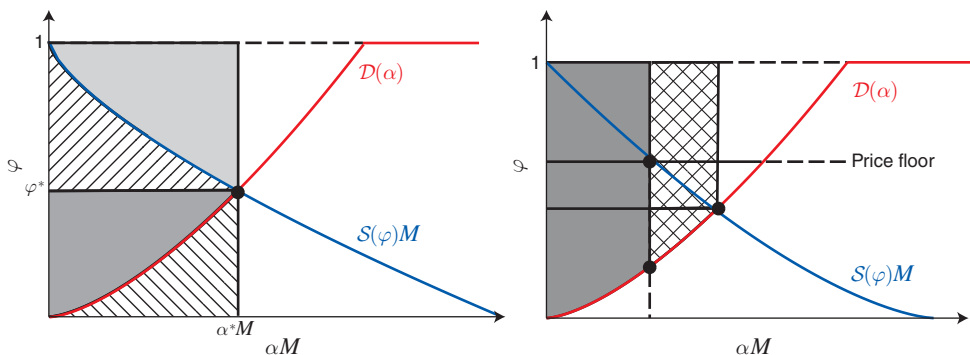


FIGURE 5. THE SOCIAL COSTS OF CRIME

Notes: The areas are measured in dollars after I scale the units on the horizontal axis by  $M$ . For visual clarity, both panels assume punishment  $\ell = 0$ , and so a failure rate ceiling  $\hat{\varphi} = 1$ . Left: The top left dashed region is criminal profits, the light-shaded region is criminal costs, and the dark-shaded region vigilance expenses. The dashed area below demand  $D(\cdot)$  is the potential victims' surplus, i.e., vigilance benefits  $\alpha^* M$  less total costs. Notice that criminal costs plus vigilance costs are strictly less than the rectangle  $\alpha^* M$ , namely, the value of potential gains of attempted crimes. The difference is the dashed areas, owing to victim and criminal heterogeneity and vigilance cost convexity. Right: Mandated vigilance resembles a price floor, and it lowers the crime rate and attempted crime rate when binding. The crosshatched area captures the reduction in total losses in response to less criminal activity, whereas the light-shaded area represents total potential victims' losses after the price floor is dictated.

the demand  $D(\alpha)$ .<sup>34</sup> In other words,  $v^* = \int_0^{\alpha^* M} [\varphi^* - D(\alpha)] d(\alpha M)$ . Because the demand curve  $D$  depicts victims' "reservation price" at any "quantity," the area over it and below  $\varphi = \varphi^*$  captures a cost, and their surplus is thus the triangular area below their demand. Next, I explore three applications.

**Rent-Seeking:** How well do the rewards from crime approximate the social costs of crime? Becker (1968) argues that in a competitive theft market, the market value of the property loss should approximate the total criminal costs (Becker 1968, p. 171, footnote 3). Along these lines, Tullock (1967) had already crystalized a more general insight, implying that the property value at stake should approximate the costs of all agents involved, namely, potential victims and criminals. On the other hand, his "Tullock paradox" (Tullock 1980) later observed that total costs are often swamped by the potential gains. As seen in the left panel of Figure 5, *the social costs of crime are strictly less than the potential gains*. This owes to the strict convexity of the vigilance costs and to potential criminal and victim heterogeneity. Indeed, it can be shown that social costs equal potential gains only in the extreme case, in which vigilance costs are linear and criminals and victims are homogeneous, forcing demand and supply to have extreme elasticities (i.e., 0 or  $\infty$ ).

**Price Floors:** Next, consider the effects of regulating vigilance technology (see, e.g., Vollaard and van Ours 2011; van Ours and Vollaard 2015). As discussed in Section VA, these interventions effectively impact the distribution of vigilance cost

<sup>34</sup> If in equilibrium some types  $\xi$  chose  $\hat{\phi} = 1$ , this area would underestimate vigilance expenses.



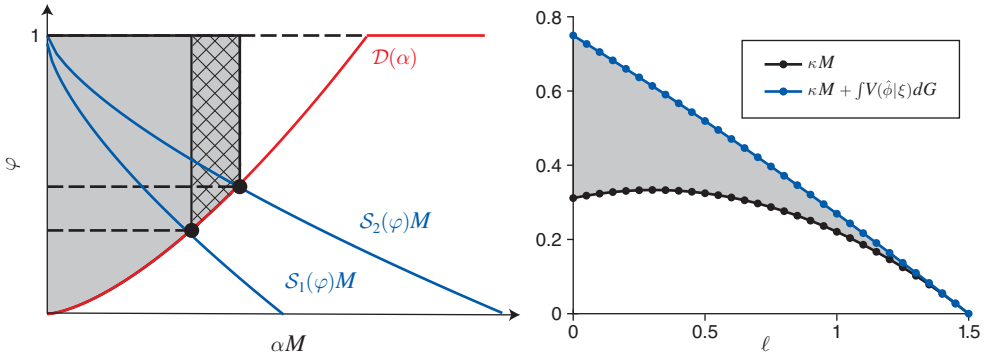


FIGURE 6. PUNISHMENT AND WELFARE

Notes: Left: Suppose that punishment  $\ell$  rises from  $\ell = 0$  so that the supply curve shifts left from  $S_2$  to  $S_1$ . While the crime rate may rise, total potential victims' losses unambiguously fall. The crosshatched area measures the decrease in potential victims' losses owed to the increase in punishment. Right: This panel considers Example 1's parameterization (with  $M = 1.5$ ). The crime rate (measured in dollars) is hump shaped in punishment  $\ell$ , whereas total potential victim losses is decreasing in  $\ell$  because of the benefits of lowering vigilance.

across potential victims. Given a fixed supply curve, these regulations can be seen as a *price floor* because they force a higher failure rate of attempted crimes, leading to fewer crimes and attempted crimes (Proposition 4). As seen in the right panel of Figure 5, these regulations reduce potential victims' losses, although they increase vigilance expenses. Unlike punishment, regulating vigilance is effective not only in reducing crime but also lowering potential victims' losses.

**Punishment:** Finally, I ask what happens to potential victims' expected losses when punishment rises? Because of the criminal Laffer curve (Section IV) the answer is unclear a priori since greater penalties reduce vigilance expenses but they could also raise expected property losses (Proposition 1). However, this puzzle is easily resolved using Figure 5. Indeed, as discussed in Section IV, as punishment  $\ell$  rises, the supply curve shifts left along the demand curve; thus, both potential victims' losses (the dark area in Figure 5's left panel plus the crime rectangle area  $\alpha^*(1 - \varphi^*)M$ ) and the net benefits of vigilance (the dashed area below  $\mathcal{D}$  in the left panel of Figure 5) must unambiguously fall. That is, *greater expected punishment decreases total potential victims' losses and the net benefits of vigilance*, even when punishment raises the crime rate; see Figure 6 (left panel). This observation highlights that the effect of punishment on crime provides an incomplete description of the impact of punishment on welfare. As seen in the right panel of Figure 6, the effect of punishment on the crime rate *understates* its impact on potential victims' losses because it ignores the benefits to potential victims from reducing vigilance spending, which is captured by the shaded area.

### VII. Minimizing Potential Victims' Losses

Criminal activity is shaped by the unobservable vigilance efforts that potential victims make to prevent losing their property. Because all potential victims face the

same attempted crime rate, independent of their vigilance choices, the more potential victims protect themselves, the lower the incentives to attempt crimes. Thus, there is a *positive externality* among potential victims, as each benefits from the others' vigilance. So by standard economic logic, *total vigilance must be underprovided*, as theoretically argued by Shavell (1991) and empirically tested by Ayres and Levitt (1998). However, a missing aspect in the literature is understanding the determinants of the *degree of underprovision*. If legal punishment were to rise, should we expect more or less underprovision of vigilance?

To this end, consider a social planner who is in charge of coordinating unobservable vigilance efforts to minimize potential victims' expected losses. Unlike potential victims, who take the attempted crime rate as given, the social planner anticipates how changes in vigilance impact the attempted crime rate. Specifically, suppose that the planner desires an average failure rate  $\varphi^o \in [0, 1]$  and so an attempted crime rate  $\alpha^o = \mathcal{S}(\varphi^o)$ . The planner could implement any  $\varphi$  by exploiting the *inverse demand function*  $\mathcal{D}^{-1} : [0, 1] \mapsto [0, \bar{\alpha}]$ , where  $\bar{\alpha}$  is the attempted crime rate ceiling (see Section III).<sup>35,36</sup> Indeed, by the FOC (3), the planner can mandate individual failure rates  $\hat{\phi}^o(\varphi|\xi)$  obeying,  $\mathcal{D}^{-1}(\varphi)M \equiv V_{\phi}(\hat{\phi}^o(\varphi|\xi)|\xi)$  for each  $\xi \in [0, 1]$ , and thus  $\int \hat{\phi}^o(\varphi|\xi) dG(\xi) = \varphi$ .<sup>37</sup> Intuitively,  $\hat{\phi}^o(\varphi|\xi)$  is the failure rate optimally chosen by potential victim  $\xi$  when the attempted crime rate is  $\mathcal{D}^{-1}(\varphi)$ . Clearly,  $\hat{\phi}^o(\cdot|\xi)$  is monotone and  $\hat{\phi}^o(0|\xi) = 0$  for all  $\xi$ . Altogether, the planner's failure rate target  $\varphi^o$  solves

$$(7) \quad \min_{\varphi \in [0,1]} \mathcal{S}(\varphi)(1 - \varphi)M + \int_0^1 V(\hat{\phi}^o(\varphi|\xi)|\xi) dG(\xi).$$

To have a convex program, I assume that the density elasticity  $\mathcal{E}_{1-\varphi}(f(\bar{c})) \geq -2$ .<sup>38</sup> Appendix A.A11 shows that the optimal failure rate  $\varphi^o \in (0, 1)$  and attempted crime rate  $\alpha^o > 0$  obey

$$(8) \quad \mathcal{S}(\varphi^o) - \mathcal{S}'(\varphi^o)(1 - \varphi^o) = \mathcal{D}^{-1}(\varphi^o)$$

and  $\alpha^o = \mathcal{S}(\varphi^o)$ , respectively. The individual failure rates obey  $\hat{\phi}^o(\varphi^o|\xi)$  for all  $\xi \in [0, 1]$ .

As seen in Figure 7, the optimal failure rate is given by the intersection of the inverse demand curve and the *marginal crime rate supply curve*  $\mathcal{MS}(\varphi) \equiv [\mathcal{S}(\varphi)(1 - \varphi)]_{\varphi}$ . The resulting attempted crime rate  $\alpha^o$  is fixed by the supply curve  $\mathcal{S}$ . Notice the analogy with noncompetitive markets, where a monopolist chooses how much to produce by equalizing marginal costs (supply) and marginal revenues, and the final price is fixed by the demand curve. The same occurs here, as potential victims are the producers of vigilance, and so for them the attempted crime rate  $\alpha$  resembles a price and the failure rate  $\varphi$  a quantity.

<sup>35</sup>This inverse function is well defined:  $\mathcal{D}(\cdot)$  is strictly increasing for all attempted crime rates  $\alpha \leq \bar{\alpha}$ .

<sup>36</sup>In words,  $\mathcal{D}^{-1}(\varphi)$  is the attempted crime rate that induces an average failure rate  $\varphi$ .

<sup>37</sup>For simplicity, I assume that the FOC (3) holds for all potential victim types  $\xi$ .

<sup>38</sup>This condition holds when the cost distribution  $F$  is either convex or not too concave.

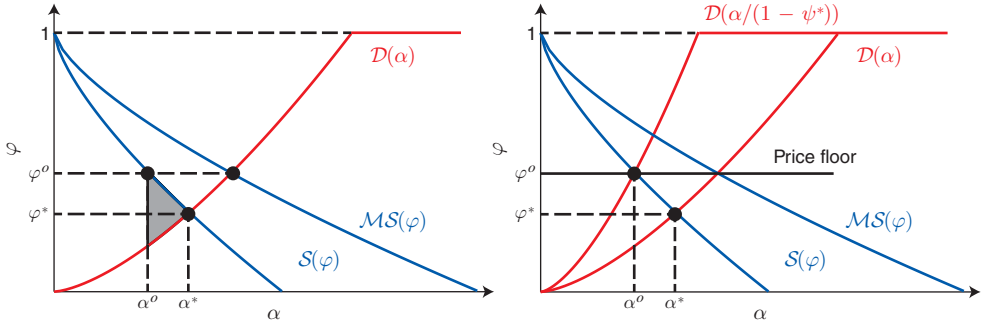


FIGURE 7. CENTRALIZING VIGILANCE

Notes: Both panels assume  $\ell = 0$ , and so  $\bar{\varphi} = 1$ . Left: The optimal failure rate is given by the intersection of the inverse demand  $D^{-1}$  and marginal supply  $MS$ , which lies above  $S$ . The respective optimal failure and attempted crime rates are greater and lower than the equilibrium ones. The shaded area captures the net benefits of centralizing vigilance. Right: A subsidy  $\psi^*$  on vigilance costs or a price floor implements the desired levels of crime and vigilance.

Compared to the decentralized case (Figure 1), the new term  $-\mathcal{S}'(\varphi^o)(1 - \varphi^o) < 0$  in (8) is the *marginal deterrence effect* of vigilance (Shavell 1991), as the planner also influences criminal entry. Since this effect is negative, *the failure rate is higher and the crime rate and the attempted crime rate are lower than in the competitive benchmark* (see the left panel of Figure 7). Also, in the same panel, the shaded triangle represents the *benefits* of coordinating vigilance efforts. While total vigilance may rise compared to the benchmark, the reduction in property losses more than compensates for it. Criminal profits unambiguously fall.

The degree of vigilance underprovision is captured by the gap  $\varphi^o - \varphi^*$ , or by the relative excess of demand, namely,  $(D^{-1} - S)/S$ , as it measures the vigilance deviation from the competitive equilibrium. Furthermore, by (8), at the optimum, the degree of underprovision is equal to the elasticity of the supply curve (with respect to the failure rate) evaluated at  $\varphi = \varphi^o$ :

$$(9) \quad \frac{D^{-1}(\varphi^o) - S(\varphi^o)}{S(\varphi^o)} = \mathcal{E}_{1-\varphi}(S).$$

The degree of underprovision is, thus, affected by the supply elasticity  $\mathcal{E}_{1-\varphi}(S)$ , which is a function of the expected punishment  $\ell$  and the criminal cost distribution  $F$ . Proposition 6 examines how punishment  $\ell$  impacts the optimal vigilance levels and the underprovision gap.

**PROPOSITION 6:** (i) If  $(c + \ell)f(c)$  falls in  $c$ , the optimal failure rate  $\varphi^o$  and the degree of vigilance underprovision both rise as punishment  $\ell$  rises; (ii) if  $(c + \ell)f(c)$  rises in  $c$ , the optimal failure rate  $\varphi^o$  falls in punishment  $\ell$ , but the degree of vigilance underprovision rises in  $\ell$ , provided the cost distribution  $F$  is log-concave and the elasticity  $cf(c)/F(c)$  rises in  $c$ .

The sufficient conditions in Proposition 6 do not depend on endogenous variables and are critically related to the curvature of the cost distribution  $F$  and the punishment level  $\ell$ . Indeed,  $(c + \ell)f(c)$  is increasing in  $c$  when  $F$  is convex or not too concave, and it is decreasing in  $c$  when  $F$  is concave and the absolute rate of change of  $f$  is high enough.

Proposition 6 highlights another channel through which penalties may have an undesirable effect. In the benchmark case, an increase in punishment unambiguously lowers the equilibrium failure rate  $\varphi^*$  (Proposition 1). However, when vigilance is centralized, the effect of punishment on  $\varphi^o$  is ambiguous: Depending on the curvature of the criminal cost distribution  $F$ , an increase in punishment  $\ell$  may lower or raise the marginal deterrence effect of vigilance, and so it may decrease or increase the optimal failure rate  $\varphi^o$ . In particular, when condition (i) in Proposition 6 holds, an increase in punishment raises the optimal failure rate  $\varphi^o$ , and it lowers the equilibrium failure rate  $\varphi^*$ . Thus, as punishment rises, the equilibrium failure rate not only falls but also becomes more distant to its desirable level.<sup>39</sup>

Finally, I briefly discuss how the planner could implement the optimal failure rate  $\varphi^o$ . This could be done by either subsidizing vigilance or mandating it with a price floor. Indeed, suppose that the government can subsidize vigilance by paying a fraction  $\psi \in (0, 1)$  of vigilance costs. Potential victim  $\xi$  expected losses would turn to  $\alpha(1 - \phi)M + (1 - \psi)V(\phi|\xi)$ , and individual demands would obey  $\hat{\phi}\left(\frac{\alpha}{1-\psi}|\xi\right) = \operatorname{argmin}_{\phi} \frac{\alpha}{1-\psi}(1 - \phi)M + V(\phi|\xi)$  for all  $\xi$ . That is, potential victims would behave *as if* they faced an attempted crime rate  $\frac{\alpha}{1-\psi} > \alpha$ . As seen in Figure 7 (right panel), the planner can implement her desired levels of crime and vigilance with a *Pigovian subsidy*  $\psi^*$  obeying  $\mathcal{D}\left(\frac{\alpha^o}{1-\psi^*}\right) = \varphi^o$ . Alternatively, she could mandate a minimum vigilance level, emulating a price floor (van Ours and Vollaard 2015).

Altogether, the government need not rely on greater punishments (as argued, their efficacy is limited), but rather its policies should be targeted to directly incentivize, or regulate, potential victims' vigilant behavior and thereby indirectly impact criminals' incentives.

**Remark 2 (Social Cost Minimization):** One could also imagine that the social planner's goal is to minimize the social costs (6). If so, the planner would elect a lower failure rate and more attempted crimes than those that minimize potential victims' losses (7). Because now the marginal cost of raising the average failure rate  $\varphi$  not only embeds total marginal vigilance expenses but also the forgone marginal criminal profits. Appendix B shows that if the supply elasticity  $\mathcal{E}_{1-\varphi}(\mathcal{S}) > 1$ , then the failure rate minimizer lies between  $\varphi^*$  and  $\varphi^o$ . However, if  $\mathcal{E}_{1-\varphi}(\mathcal{S}) < 1$ , the social cost minimizer entails a lower failure rate and more attempted crimes compared to the equilibrium levels. In other words, vigilance is overprovided in equilibrium. Proposition B.1 then provides sufficient conditions for either case in

<sup>39</sup> Appendix A.A12 shows that if condition (ii) holds, the relative excess of demand (9) rises in punishment  $\ell$ .

terms of primitives. The former case arises when the criminal cost distribution  $F$  is not “too concave,” while the latter one when  $F$  is “sufficiently” concave.<sup>40</sup>

### VIII. The Observable Vigilance Case

In this section, I relax the assumption that vigilance is unobservable. If potential criminals can observe potential victims’ vigilance levels, then a new criminal decision margin emerges: criminals must choose, e.g., whether to burgle the gated mansion or the ungated one. That is, when vigilance is observable, the interaction between potential victims and criminals become *sequential*. First, potential victims simultaneously and independently choose their *vigilance level*  $v \geq 0$ . Then, potential criminals observe the vigilance choices made by potential victims and simultaneously and independently choose where to target their offense, if any.

Crucially, the observability of vigilance changes the strategic nature of crime. Indeed, when vigilance is observable, a novel *signaling* force arises, as potential victims’ types  $\xi$  are private information. Because criminals care about the failure rate of their attempted crimes, observing vigilance  $v$  conveys information about types  $\xi$  and so about the expected failure rate associated with vigilance  $v$ .<sup>41</sup> To encompass this sequentiality, the equilibrium notion, used throughout this paper, must naturally change from Nash equilibrium to perfect equilibrium. In a perfect equilibrium, potential criminals use the information contained in vigilance choices to make failure rate inferences and target their offenses, whereas potential victims understand this when choosing their vigilance intensities. Henceforth, I examine *separating* equilibria in which vigilance effectively varies across potential victims.

For the sake of clarity, I now introduce some notations that are only necessary here. As discussed in Section II, the failure rate of an attempted crime depends on both vigilance and the (privately known) potential victim’s type. Specifically, for each type  $\xi \in [0, 1]$  and vigilance  $v \geq 0$ , the induced *failure rate function*  $\tilde{\phi}(v, \xi) \in [0, 1]$  obeys  $v \equiv V(\tilde{\phi}(v, \xi) | \xi)$ . It is increasing in vigilance  $v$  (as  $V_\phi > 0$ ) and decreasing in  $\xi$  (as also  $V_\xi > 0$ ) with partials

$$(10) \quad \tilde{\phi}_v(v, \xi) = 1/V_\phi(\phi(v, \xi) | \xi)$$

and

$$\tilde{\phi}_\xi(v, \xi) = -V_\xi(\phi(v, \xi) | \xi)/V_\phi(\phi(v, \xi) | \xi).$$

Also,  $\tilde{\phi}_{v\xi}(v, \xi) \leq 0$  (Claim C.1), so vigilance’s efficacy and marginal efficacy fall as  $\xi$  rises.

Now, for every observed vigilance  $v \geq 0$ , the *attempted crime rate function*  $\tilde{\alpha}(v, \hat{\xi}) \geq 0$  is the attempted crime rate aimed to vigilance  $v$  when the *criminal inference function*,  $\hat{\xi}_I(\cdot)$ , about the victims’ type obeys  $\hat{\xi}_I(v) = \hat{\xi} \in [0, 1]$ .

<sup>40</sup>See propositions 3A and 3B in Shavell (1991) for a related discussion.

<sup>41</sup>Baumann and Friehe (2013) study a model with observable vigilance in which victims have private information. They consider *binary* vigilance and a *fixed* mass of criminals, which simplifies the analysis.

I assume  $\tilde{\alpha}$  is differentiable with nonvanishing partials,  $\tilde{\alpha}_v \neq 0$  and  $\tilde{\alpha}_{\xi} \neq 0$ . Slightly abusing notation, a potential victim's losses are  $\mathcal{L}(\xi, \hat{\xi}, v | \tilde{\alpha}) \equiv \tilde{\alpha}(v, \hat{\xi}) (1 - \tilde{\phi}(v, \xi))M + v$ . These are the losses from crime for potential victim  $\xi \in [0, 1]$  who chooses vigilance  $v \geq 0$  when the realized criminal inference is  $\hat{\xi} \in [0, 1]$ .

Next, consider an equilibrium candidate *vigilance function*  $\chi(\cdot)$  for potential victims, where  $\chi(\xi) \geq 0$  is the vigilance level exerted by potential victim  $\xi \in [0, 1]$ . In a separating equilibrium,  $\chi$  must be *one-to-one*, and so invertible. Thus, on the equilibrium path, i.e., if a potential victim chooses  $v \in \chi([0, 1])$ , then, irrespective of her true type, potential criminals infer that the victim's type is  $\hat{\xi}_I(v) \equiv \chi^{-1}(v)$  and the induced failure rate is  $\tilde{\phi}(v, \chi^{-1}(v))$ . Consequently, the total amount of attempted crimes obeys  $\int_{\chi([0,1])} \tilde{\alpha}(v, \chi^{-1}(v)) dv$ . Now, off the equilibrium path, i.e., for  $v \notin \chi([0, 1])$ , equilibrium perfection puts no restriction on the inference performed by criminals as long as  $\hat{\xi}_I(v) \in [0, 1]$ .

In equilibrium, the vigilance function  $\chi(\cdot)$  must incentivize potential criminals to target their attempted crimes following  $\tilde{\alpha}(\cdot, \chi^{-1}(\cdot))$  on path. Likewise, the attempted crime rate function  $\tilde{\alpha}(\cdot, \hat{\xi}_I(\cdot))$  must encourage potential victims to follow the vigilance function  $\chi(\cdot)$ .

Formally, a *separating equilibrium* is a pair  $(\chi(\cdot), \tilde{\alpha}(\cdot))$  such that (i) given the vigilance function  $\chi(\cdot)$ , criminal profit maximization yields attempted crime rates  $\tilde{\alpha}(v, \chi^{-1}(v))$  for  $v \in \chi([0, 1])$ , and  $\tilde{\alpha}(v, \hat{\xi})$  for  $v \notin \chi([0, 1])$  and some  $\hat{\xi}_I(v) = \hat{\xi}$ ; (ii) given the attempted crime rate function  $\tilde{\alpha}(\cdot)$ , vigilance  $\chi(\xi)$  minimizes expected losses for potential victim  $\xi \in [0, 1]$ .

Given  $\chi(\cdot)$  and  $\hat{\xi}_I$ , define  $\mathcal{L}^d(\xi, \hat{\xi}_I) \equiv \inf \{ \mathcal{L}(\xi, \hat{\xi}_I(v), v | \tilde{\alpha}) : v < \chi(0) \}$ . The next result states the necessary conditions to sustain an equilibrium. Appendix C.C2 examines when such conditions are also sufficient; see Proposition C.1.

**PROPOSITION 7:** *Consider an equilibrium  $(\chi(\cdot), \tilde{\alpha}(\cdot))$ . Then, there exists an interior failure rate  $\tilde{\varphi} \in (0, 1)$  with  $\chi(\xi) \equiv V(\tilde{\varphi} | \xi)$ , and  $\tilde{\alpha}(\chi(\xi), \xi)$  solving*

$$(11) \quad \left[ \tilde{\alpha}_v(\chi(\xi), \xi) + \tilde{\alpha}_{\xi}(\chi(\xi), \xi) \frac{1}{\chi'(\xi)} \right] V_{\phi}(\tilde{\varphi} | \xi) (1 - \tilde{\varphi})M - \tilde{\alpha}(\chi(\xi), \xi)M + V_{\phi}(\tilde{\varphi} | \xi) = 0,$$

*and market clearing:  $\int_0^1 \tilde{\alpha}(\chi(\xi), \xi) dG(\xi) = S(\tilde{\varphi})$ . The vigilance function  $\chi(\xi)$  is strictly increasing, whereas the attempted crime rate  $\tilde{\alpha}(\chi(\xi), \xi)$  is strictly decreasing, vanishing when  $\xi = 1$ . The equilibrium losses  $\mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha}) \leq \mathcal{L}^d(\xi, \hat{\xi}_I)$  for all  $\xi$  and some inference  $\hat{\xi}_I$ .*

Proposition 7 installs a constraint on the shape that any equilibrium can take. First, because each criminal optimally attempts a crime toward potential victims for whom the failure rate is the lowest, in equilibrium, the induced failure rates  $\tilde{\phi}(\chi(\xi), \xi)$  must be constant across all types  $\xi$ . Consequently, from the criminals' viewpoint, all vigilance levels are equally desirable targets. The vigilance function  $\chi$  is thus characterized by an *average failure rate*  $\tilde{\varphi}$  obeying  $\tilde{\phi}(\chi(\xi), \xi) \equiv \tilde{\varphi}$ , or



$\chi(\xi) \equiv V(\tilde{\varphi}|\xi)$ , and so *high types are more vigilant than low types* since  $\chi'(\xi) = V_\xi(\tilde{\varphi}|\xi) > 0$ . Second, given a common failure rate  $\varphi$ , the amount of attempted crimes is determined by the supply  $\mathcal{S}(\varphi)$ , as in Section III. Since  $\tilde{\alpha}(\chi(\xi), \xi)$  is the attempted crime rate faced by victim  $\xi$ , the failure rate  $\tilde{\varphi}$  must also *clear the market*:

$$(12) \quad \int_0^1 \tilde{\alpha}(\chi(\xi), \xi) dG(\xi) = \mathcal{S}(\tilde{\varphi}).$$

Third, the differential equation (11) is a consequence of potential victim optimality and equilibrium considerations. Notice that, on path, the losses of a type  $\xi$  that chooses  $v$  are  $\mathcal{L}(\xi, \chi^{-1}(v), v|\tilde{\alpha})$ . Thus, in equilibrium,  $\chi$  must be *incentive compatible*, i.e.,  $\chi(\xi)$  must solve

$$(13) \quad \min_{v \in \chi([0,1])} \tilde{\alpha}(v, \chi^{-1}(v))(1 - \tilde{\varphi}(v, \xi))M + v.$$

The first-order condition, evaluated at  $v = \chi(\xi)$  with  $\tilde{\varphi}(\chi(\xi), \xi) \equiv \tilde{\varphi}$ , yields (11), whereas the second-order condition implies the strict monotonicity of the attempted crime rate function  $\tilde{\alpha}(\chi(\cdot), \cdot)$ . In addition,  $\tilde{\alpha}(\chi(1), 1)$  must vanish; for if not, victim type  $\xi = 1$  could slightly raise her vigilance and lower her expected losses, for any off path inference; see Appendix C.1.

Finally, potential victims must have no incentives to elect vigilance  $v \notin \chi([0, 1])$ . Consider an upward deviation  $v > \chi(1)$ . Because  $\tilde{\alpha}(\chi(1), 1) = 0$ , incentive compatibility (13) makes such deviations unattractive, as they lead to losses  $\mathcal{L} \geq \chi(1)$ . Now, take a downward deviation  $v < \chi(0)$ . Such vigilance would attract all criminals, as  $\tilde{\varphi}(v, \hat{\xi}) < \tilde{\varphi}(\chi(0), \hat{\xi}) \leq \tilde{\varphi}(\chi(0), 0) = \tilde{\varphi}$ , for any off-path inference  $\hat{\xi}_I(v) = \hat{\xi}$ , yielding an attempted crime rate function  $\tilde{\alpha}(v, \hat{\xi}) \equiv \mathcal{S}(\tilde{\varphi}(v, \hat{\xi}))$ .<sup>42</sup> The *lowest downward-deviation losses* are  $\mathcal{L}^d(\xi, \hat{\xi}_I)$ , and so for some off-path inference  $\hat{\xi}_I$ , the equilibrium losses must obey  $\mathcal{L}(\xi, \xi, \chi(\xi)|\tilde{\alpha}) \leq \mathcal{L}^d(\xi, \hat{\xi}_I)$ .

Notice the qualitative differences between observable and unobservable vigilance. In the observable case, since the attempted crime rate  $\tilde{\alpha}(\chi(\xi), \xi)$  is strictly decreasing and the failure rate  $\tilde{\varphi}$  is constant, the crime rate  $\tilde{\alpha}(\chi(\xi), \xi)(1 - \tilde{\varphi})$  falls as  $\xi$  rises. By contrast, when vigilance is unobservable, potential victims face the same attempted crime rate  $\alpha^*$  but secure distinct failure rates  $\hat{\varphi}(\alpha^*|\xi)$ ; in fact, the crime rate  $\alpha^*(1 - \hat{\varphi}(\alpha^*|\xi))$  rises as type  $\xi$  rises since high types are less vigilant, i.e.,  $\hat{\varphi}_\xi < 0$ ; see Section III. This qualitative difference follows from the first-order conditions (3) and (11), for in the unobservable case the marginal cost of vigilance is *constant* across types, while it varies when vigilance is observable. Also, by Proposition 7 and Claim A.1, in the observable and unobservable cases, high-cost victims (high  $\xi$ ) exert more and less vigilance, respectively. Still, in any case, equilibrium losses  $\mathcal{L}$  are higher for high-cost victims (see Claim C.2 and Section III), but each owing to different economic trade-offs.

The observability of vigilance gives rise to new forces. As in Section VII, the first term in (11) captures the *marginal deterrence effect of vigilance for victim*  $\xi$ . By Proposition 7, this effect becomes absolutely stronger as potential victims' types  $\xi$

<sup>42</sup>Hence, the downward deviation losses for type  $\xi$  obey  $\mathcal{L}(\xi, \hat{\xi}, v|\tilde{\alpha}) = \mathcal{S}(\tilde{\varphi}(v, \hat{\xi}))(1 - \tilde{\varphi}(v, \xi))M + v$ .

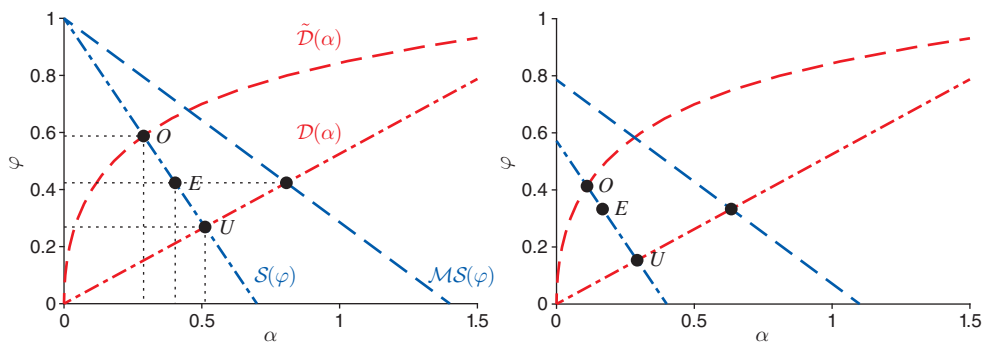


FIGURE 8. EQUILIBRIUM WITH OBSERVED VIGILANCE

Notes: Both panels consider the parameterization given in Example 1 with property value  $M = 0.7$ . Left: This panel assumes punishment  $\ell = 0$ . Points  $O$  and  $U$  denote the equilibrium with observed and unobserved vigilance, respectively, whereas  $E$  describes the levels of crime and vigilance that minimize potential victims' losses in Section VII. Right: This panel considers an increase in penalties to  $\ell = 0.3$ . Compared to the left panel, the degree of underprovision of unobserved vigilance (distance  $OE$ ) rises, as predicted by Proposition 6, whereas the degree of overprovision of observed vigilance (distance  $EU$ ) falls.

rise—namely, *high-cost victims benefit more from the observability of vigilance*.<sup>43</sup> Also, notice that the marginal deterrence effect is proportional to the decline in the attempted crime rate, aimed to victim  $\xi = \chi^{-1}(v)$ , caused by a marginal increase in vigilance  $v$ ,  $d\tilde{\alpha}/dv|_{v=\chi(\xi)}$ . In particular, the effect is determined by two additive forces. First, there is a *redirection effect* (proportional to  $\tilde{\alpha}_v$ ) as criminals divert their attempted crime toward other victims in response to an increase in vigilance.<sup>44</sup> And second, there is *learning effect* (proportional to  $\tilde{\alpha}_\xi(1/\chi')$ ), owed to criminals updating their inferences and, consequently, their victim targets.

As in Section III, the equilibrium is fixed by the intersection of supply and demand, where the demand now should be seen as a derived demand for attempted crime. Indeed, given a failure rate  $\varphi$ , the map  $\mathcal{A}_D(\varphi) \equiv \int_0^1 \tilde{\alpha} dG$  captures the induced *demand for attempted crime*, as  $\tilde{\alpha}$  induces a vigilance behavior governed by  $\chi$ , given (11). By market clearing (12), the equilibrium is fixed by the intersection of supply  $S(\cdot)$  and demand  $\mathcal{A}_D(\cdot)$ . So, the Lafferian effects, identified in Section IV, are likely to apply to this case, by common logic.

To close this section, Figure 8 simulates the model using Example 1 in order to shed further light into the potential effects of observed vigilance. As seen in the left panel,  $\mathcal{A}_D(\varphi)$  rises in  $\varphi$  and induces a demand for vigilance  $\tilde{D}(\alpha)$ , where  $\mathcal{A}_D(\tilde{D}(\alpha)) \equiv \alpha$ . More crucially,  $\tilde{D}(\cdot)$  is above  $D(\cdot)$ , and thus the equilibrium with observed vigilance (point  $O$ ) entails a *greater failure rate*  $\varphi$  and *fewer attempted crimes*  $\alpha$  and *so fewer actual crimes*  $\kappa = (1 - \varphi)\alpha$ , compared to the equilibrium with unobserved vigilance (point  $U$ ). Additionally, unlike the unobserved vigilance case, there is *overprovision* of vigilance relative to the benchmark studied in Section VII (compare points  $O$  and  $E$  in Figure 8). Finally, the right panel of Figure 8 depicts

<sup>43</sup> Consider the FOC (11). When  $\xi$  rises, the middle and right terms  $-\tilde{\alpha}M$  and  $V_\varphi$ , respectively, rise (given Proposition 7 and  $V_{\varphi\xi} \geq 0$ ). Therefore, the left term, namely, the marginal deterrence effect of vigilance, must fall in order to keep condition (11) balanced. As a result,  $d\tilde{\alpha}/dv$  cannot fall too rapidly.

<sup>44</sup> See, e.g., Gonzalez-Navarro (2013) for an empirical analysis of this diversion effect in auto theft.

the effects of punishment  $\ell$ . Since the cost distribution  $F(c) = c$  in Example 1, an increase in punishment  $\ell$  raises the degree of underprovision of unobserved vigilance (Proposition 6(ii)); however, *the degree of overprovision of observed vigilance falls as punishment rises* (distance between  $O$  and  $E$ ). This exercise suggests that when vigilance is observed, although there could be too much vigilance compared to the levels that minimize potential victims' losses, this divergence could be ameliorated with harsher punishment, as potential victims' incentives relax. A general analysis is pursued in ongoing work.

### IX. Concluding Remarks

Since the domestic security changes made post-9/11, it is more clear than ever that the costs of crime are not just the actual losses of individuals but also the vigilance expenses for crimes that never happen. This paper has examined the interplay between crime and vigilance. To this end, I developed an economic framework with both heterogeneous potential criminals and potential victims in which (1) pairwise matching of criminals and potential victims produces attempted crimes; (2) not all attempted crimes succeed; and (3) the failure of an attempted crime is probabilistic, rising as the individual vigilance level rises.

From an economic perspective, the equilibrium analysis uncovered and characterized a *vigilance force* that naturally arises in these settings. This force invariably limits the efficacy of policies aimed directly at alleviating crime, such as raising an offense's legal punishment—because victims and criminals both respond when change befalls either party. This force becomes stronger when vigilance expenses are greater than property losses, unveiling a *criminal Laffer curve*, namely, a crime rate that is hump shaped in legal punishment. In terms of welfare, the focus on crowding out effects helps to sharpen policy intervention: if the goal is to lower the crime rate or the attempted crime rate, policies aimed at incentivizing vigilance, such as mandating or subsidizing it, could be more effective than raising punishment. In fact, raising punishment may shift vigilance even further away from its efficient level.

Next, I turn to discuss some empirical and theoretical avenues for future research. First, this paper suggests that the effects of punishment on crime can be confounded, and thus disentangling crimes and attempted crimes may be a fruitful exercise. For instance, the empirical literature has found the elasticity of crime to police manpower to be in a range of  $-0.1$  to  $-2$  (Chalfin and McCrary 2017). Yet this finding need not imply that the criminal Laffer curve slopes down. Indeed, to test whether this curve slopes up or down, one needs first to differentiate attempted crimes and actual crimes and then examine the impact of punishment on both the attempted crime rate and actual crime rate.

Second, this paper predicts that the relationship between crime and its stakes is nonmonotone. Draca, Koutmeridis, and Machin (2019) recently examined the relationship between crime and the price of goods, accounting only for time-invariant vigilance such as security factors inherent to a good. These factors are embedded in the goods' fixed effects. It would be interesting to also account for varying vigilance and test whether crimes and attempted crimes rise or fall as the price of goods rises.

Varying vigilance factors may be obtained from the differential patterns of theft across households that have similar composition of goods. This could be examined using household-level and crime victimization data.

Third, in terms of identification, empirical analyses of traditional markets suggest that, when prices and quantities are observable for a given good, supply and demand can be respectively identified by exogenous demand shifters and exogenous supply shifters. This insight can be used to empirically examine the supply and demand framework that this paper provides. Indeed, if one measures the attempted crime rate  $\alpha$  and crime rate  $\kappa$ , then the failure rate  $\varphi$  could be recovered using the identity  $\kappa = (1 - \varphi)\alpha$ .<sup>45</sup> Having data  $(\alpha, \varphi)$  can be then used to guide the identification of the supply's and demand's slopes. In particular, as seen in Sections IV–V, changes in punishment or policing shift the supply curve  $\mathcal{S}$  along the demand  $\mathcal{D}$ , while changes in vigilance technology shift the demand curve  $\mathcal{D}$  along the supply  $\mathcal{S}$ . Identifying the elasticity of the supply and demand curves is useful to inform policy. For instance, the demand elasticity is key for understanding the crowding out effects of punishment, whereas the supply elasticity determines the inefficiency gap outlined in Section VII.

Finally, from a theoretical angle, in this paper optimal public responses to crime are unmodeled in order to isolate the effects of vigilance on crime. Modeling both endogenous public and private responses to crime would shed light on the classic debate of public versus private law enforcement (Becker and Stigler 1974; Landes and Posner 1975). Also, this paper separately examines the cases of unobserved and observed vigilance. It would be interesting to augment the model and allow potential victims to choose whether or not to disclose their vigilance, as in Baumann, Denter, and Friehe (2019). These avenues are left for future research.

## APPENDIX A. PRELIMINARY RESULTS AND OMITTED PROOFS

### A1. Optimal Vigilance Responses

**CLAIM A.1:** Consider an attempted crime rate  $\alpha < V_\phi(1|1)/M$ . If  $\alpha > V_\phi(1|0)/M$ , vigilance expenditures  $V(\hat{\phi}(\alpha|\xi)|\xi)$  are hump shaped in type  $\xi$ ; otherwise, they are decreasing in  $\xi$ .

**PROOF:**

Fix  $\alpha$ , and call  $\xi_h \geq 0$  the first type  $\xi \in [0, 1]$  for which  $V_\phi(1|\xi)/M \geq \alpha$ . If  $\xi_h > 0$ , then, by (3), all types  $\xi < \xi_h$  choose  $\hat{\phi}(\alpha|\xi) = 1$  (for  $V_\phi(1|\xi)/M > \alpha$ ), whereas types  $\xi \geq \xi_h$  elect  $\hat{\phi}(\alpha|\xi) \in (0, 1)$  obeying (3). Thus, for  $\xi < \xi_h$ , vigilance expenditures,  $V(1|\xi)$ , increase in  $\xi$  since  $V_\xi \geq 0$ . However, for  $\xi \geq \xi_h$ , differentiating (3) yields  $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi} \leq 0$ , and

$$\frac{dV(\hat{\phi}|\xi)}{d\xi} = V_\phi \hat{\phi}_\xi + V_\xi = V_\xi - V_\phi \frac{V_{\phi\xi}}{V_{\phi\phi}} = \frac{V_\phi V_\xi}{V_{\phi\phi}} \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{V_\phi}{V} + \frac{V_\phi}{V} - \frac{V_{\phi\xi}}{V_\xi} \right) \leq 0,$$

<sup>45</sup>For instance, in motor vehicle theft, one could connect the model to the FBI data by letting  $\alpha$  be simply the reported theft rate and  $\kappa$  the proportion of those that are not cleared with an arrest. In general, the mapping of the model to the data will depend on the specific property crime in mind.

where the inequality holds since  $V$  is log-concave in  $\phi$  and log-supermodular in  $(\phi, \xi)$ , and also  $V_\xi, V_\phi, V_{\phi\phi} \geq 0$ . Altogether, vigilance  $V(\hat{\phi}|\xi)$  rises for  $\xi < \xi_h$  and falls for  $\xi \geq \xi_h$ .

Finally, notice that  $\xi_h > 0$  for  $\alpha > V_\phi(1|0)/M$ ; otherwise,  $\xi_h = 0$ , and so vigilance expenditures  $V(\hat{\phi}|\xi)$  decrease in  $\xi \in [0, 1]$ , as previously argued. ■

### A2. The Demand Elasticity

I now show that demand  $\mathcal{D}$  is less elastic with  $G_H$  than  $G_L$  if  $g_H(\xi)/g_L(\xi)$  is increasing. To this end, consider the following parameterized type-distribution  $H: [0, 1] \times \{0, 1\} \rightarrow [0, 1]$  with  $(\xi, \theta) \mapsto H$  and density  $h(\xi, \theta)$ . Also, let  $H(\xi, 0) \equiv G_L(\xi)$  and  $H(\xi, 1) \equiv G_H(\xi)$ .

**Step 1:**  $h(\xi, \theta)$  IS LOG-SUPERMODULAR IN  $(\xi, \theta)$ . To see this, take any  $\xi' \geq \xi$ , then  $g_H(\xi')/g_L(\xi') \geq g_H(\xi)/g_L(\xi)$ , which rearranging terms yields  $g_H(\xi')g_L(\xi) \geq g_H(\xi)g_L(\xi')$ , which is equivalent to  $h(\xi', 1)h(\xi, 0) \geq h(\xi', 0)h(\xi, 1)$ .

**Step 2:**  $\hat{\phi}(\alpha, \xi)$  IS LOG-SUBMODULAR IN  $(\alpha, \xi)$ . Log-differentiate the  $\alpha$  derivative  $M = V_{\phi\phi}\hat{\phi}_\alpha$  of the FOC (3) in  $\xi$  to get

$$(A1) \quad 0 = \frac{V_{\phi\phi\phi}\hat{\phi}_\xi + V_{\phi\phi\xi}}{V_{\phi\phi}} + \frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} \Rightarrow \frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} = \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \cdot \frac{V_{\phi\xi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\phi}}$$

$$= \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} \right),$$

where I used  $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi}$ , found by differentiating (3). Since  $\alpha\hat{\phi}_\alpha = V_\phi/V_{\phi\phi} > 0$  by (3),

$$\frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} - \frac{\hat{\phi}_\xi}{\hat{\phi}} \leq 0 \Leftrightarrow \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} + \frac{1}{\hat{\phi}} \right) \leq 0.$$

This last inequality holds, because the parenthesized term is negative. Indeed,

$$\frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} + \frac{1}{\hat{\phi}} = \left( \frac{1}{\hat{\phi}} + \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} \right) + \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} \right) \leq 0,$$

because the first parenthesized expression is negative since  $\phi V_{\phi\phi}/V_\phi$  is nonincreasing in  $\phi$ , whereas the second parenthesized term is negative since  $V_\phi$  is log-supermodular in  $(\phi, \xi)$ , by assumption. Since  $V_{\phi\xi} > 0$  and  $V_{\phi\phi} > 0$ , we have that  $\hat{\phi}$  is log-submodular in  $(\alpha, \xi)$ .

**Step 3:**  $\mathcal{D}(\alpha, \theta) = \int \hat{\phi}(\alpha, \xi)h(\xi, \theta) d\xi$  IS LOG-SUBMODULAR IN  $(\alpha, \theta)$ . Fix  $\alpha > 0$ . Since  $\hat{\phi}$  is log-submodular in  $(\alpha, \xi)$ , we have  $\hat{\phi}(\alpha|\xi)/\hat{\phi}_\alpha(\alpha|\xi)$  is

increasing in  $\xi$ . By lemma 4 in Athey (2002), since  $h(\xi, \theta)$  is log-supermodular in  $(\xi, \theta)$ , we have that

$$\frac{\mathcal{D}(\alpha, \theta)}{\mathcal{D}_\alpha(\alpha, \theta)} = \frac{\int \hat{\phi}(\alpha | \xi) h(\xi, \theta) d\xi}{\int \hat{\phi}_\alpha(\alpha | \xi) h(\xi, \theta) d\xi}$$

is increasing in  $\theta$ , which implies that  $\alpha \mathcal{D}_\alpha / \mathcal{D}$  is decreasing in  $\theta$ . This means that the market demand  $\mathcal{D}(\alpha, \theta)$  is log-submodular in  $(\alpha, \theta)$ , or  $\alpha \mathcal{D}_\alpha(\alpha, 1) / \mathcal{D}(\alpha, 1) \leq \alpha \mathcal{D}_\alpha(\alpha, 0) / \mathcal{D}(\alpha, 0)$ , or the demand  $\mathcal{D}$  is less elastic with  $G_H$  than  $G_L$ . ■

### A3. The Supply Elasticity

Paralleling Section A.A2, I now show that *supply  $\mathcal{S}$  is less elastic with  $F_H$  than  $F_L$  if  $f_H(c) / f_L(c)$  is increasing*. Consider the auxiliary and parameterized cost distribution  $K: \mathbb{R}_+ \times \{0, 1\} \rightarrow [0, \infty]$  with  $(c, \theta) \mapsto K$  and density  $k(c, \theta)$ . Also, let  $K(c, 0) \equiv F_L(c)$  and  $K(c, 1) \equiv F_H(c)$ . Following the same logic,  $k(c, \theta)$  is log-supermodular in  $(c, \theta)$  since the likelihood ratio  $k(c, 1) / k(c, 0)$  is monotone. Thus, the cost distribution  $K(c, \theta) = \int \mathbf{1}_{[0, c]}(c') k(c', \theta) dc'$  is log-supermodular in  $(c, \theta)$  since the indicator  $\mathbf{1}_{[0, c]}(c')$  is log-supermodular in  $(c, c')$ ; see lemmas 3–4 in Athey (2002). Altogether,  $k(c, \theta) / K(c, \theta)$  is increasing in  $\theta$ , or

$$k(c, 1) / K(c, 1) = f_H(c) / F_H(c) \geq f_L(c) / F_L(c) = k(c, 0) / K(c, 0).$$

Next, using (5) with the distribution  $K$ , the supply elasticity is given by

$$\mathcal{E}_\varphi(\mathcal{S}) = -\frac{k(\bar{c}, \theta)}{K(\bar{c}, \theta)} \left( \frac{M - \ell - \bar{c}}{\bar{c}} \right),$$

where  $\bar{c}$  is the marginal potential criminal. Since the parenthesized term above is positive, the supply is less elastic with  $\theta = 1$  or  $F_H$  than  $F_L$ . ■

### A4. Existence and Uniqueness of Equilibrium

First, since the demand  $\mathcal{D}(\alpha)$  is strictly increasing in  $\alpha$  for  $\alpha \leq \bar{\alpha}$ , one can define the inverse demand map  $\varphi \mapsto \mathcal{D}^{-1}(\varphi) \in [0, \bar{\alpha}]$ . Next, define the *excess of supply*  $\mathcal{ES}(\varphi) \equiv \mathcal{S}(\varphi) - \mathcal{D}^{-1}(\varphi)$ . Clearly, when the failure rate is perfect,  $\mathcal{S}(1) = 0 < \mathcal{D}^{-1}(1) = \bar{\alpha}$ . Also, since the demand  $\mathcal{D}(0) = 0$ , its inverse obeys  $\mathcal{D}^{-1}(0) = 0 < \mathcal{S}(0) = F(M - \ell)$ . Altogether, the excess of supply satisfies:  $\mathcal{ES}(1) < 0 < \mathcal{ES}(0)$ . Thus, by the Intermediate Value Theorem (IVT), there exists  $\varphi^* \in (0, 1)$  such that the excess of supply vanishes  $\mathcal{ES}(\varphi^*) = 0$ , or  $\mathcal{S}(\varphi^*) = \mathcal{D}^{-1}(\varphi^*)$ . Finally, since supply falls in the failure rate, while the inverse demand rises in it,  $\varphi^*$  is unique because the excess of supply  $\mathcal{ES}(\varphi)$  strictly falls in  $\varphi$ . ■

### A5. The Criminal Laffer Curve: Proof of Proposition 1

LEMMA A.1: *Along the demand curve, the crime rate  $\alpha(1 - \mathcal{D}(\alpha))$  is hump shaped.*



I prove this result in steps. Define the *individual crime rate function*  $\bar{\kappa}(\alpha, \xi) \equiv \alpha(1 - \hat{\phi}(\alpha, \xi))$  for victim  $\xi$ , given  $\alpha$ . Notice that  $\bar{\kappa}(0, \xi) = 0$ , and also  $\bar{\kappa}(\alpha, \xi) = 0$  for all  $\alpha \geq \bar{\alpha}_\xi$ , where  $\bar{\alpha}_\xi \equiv V_\phi(1|\xi)/M$  is the *attempted crime rate ceiling for victim*  $\xi$ . Also, the *critical attempted crime rate*  $\hat{\alpha}_\xi \in (0, \bar{\alpha}_\xi)$  is the one that maximizes the crime rate  $\bar{\kappa}(\alpha, \xi)$ , for all  $\xi$ .

**Step 1: THE INDIVIDUAL CRIME RATE IS QUASI-CONCAVE IN  $\alpha$ .** Consider a specific potential victim  $\xi$ . Now, think of  $\bar{\kappa}(\alpha|\xi)M$  as the total criminal revenue and  $(1 - \hat{\phi}(\alpha|\xi))M$  as the price. Then, the individual crime rate  $\bar{\kappa}$  falls in  $\alpha$  along the demand curve if  $(1 - \hat{\phi}(\alpha|\xi))\alpha M$  rises in  $(1 - \hat{\phi}(\alpha|\xi))M$ . Since the elasticity of a product is the sum of the elasticities,

$$\begin{aligned} (A2) \quad \mathcal{E}_{(1-\hat{\phi}(\alpha|\xi))M}((1 - \hat{\phi}(\alpha|\xi))\alpha M) &= 1 + \mathcal{E}_{(1-\hat{\phi}(\alpha|\xi))M}(\alpha) \\ &= 1 - (1 - \hat{\phi}(\alpha|\xi)) / (\alpha \hat{\phi}_\alpha(\alpha|\xi)) \\ &> 0. \end{aligned}$$

Next, I show that the left side of (A2) is increasing in  $\alpha$  and respectively negative and positive for low and high values of  $\alpha$ . For the monotonicity,  $(1 - \hat{\phi}(\alpha|\xi))$  is decreasing in  $\alpha$  since the failure rate  $\hat{\phi}(\alpha|\xi)$  is increasing in  $\alpha$ . Now, I claim that  $\alpha \hat{\phi}_\alpha(\alpha|\xi)$  is increasing in  $\alpha$ . Indeed, using (3),  $\alpha \hat{\phi}_\alpha(\alpha|\xi) = V_\phi(\hat{\phi}|\xi) / V_{\phi\phi}(\hat{\phi}|\xi)$ . Since for each type  $\xi$ , marginal costs  $V_\phi$  are log-concave,  $V_\phi(\hat{\phi}|\xi) / V_{\phi\phi}(\hat{\phi}|\xi)$  is monotone in  $\hat{\phi}$ , and so in  $\alpha$ .

Hence, the left side of (A2) rises in  $\alpha$ . Also, it is positive at  $\alpha = V_\phi(1|\xi)/M$  since  $\hat{\phi}(\alpha|\xi) = 1$  and negative as  $\alpha \downarrow 0$  since  $\hat{\phi}(0|\xi) = 0$  and  $\lim_{\phi \downarrow 0} V_\phi / V_{\phi\phi} < 1$ , so that  $\lim_{\alpha \downarrow 0} \alpha \hat{\phi}_\alpha(\alpha|\xi) < 1$ . So, by continuity, the left side of (A2) is positive for  $\alpha > \hat{\alpha}_\xi$  and negative for  $\alpha < \hat{\alpha}_\xi$ , for some  $\hat{\alpha}_\xi \in (0, V_\phi(1|\xi)/M)$ . That is,  $\bar{\kappa}(\alpha|\xi)$  is hump shaped in  $\alpha$ , peaking at  $\hat{\alpha}_\xi$ . So,  $\bar{\kappa}$  is quasi-concave in  $\alpha$  since it is hump shaped in  $\alpha < \bar{\alpha}_\xi$  and then zero for  $\alpha \geq \bar{\alpha}_\xi$ . ■

**Step 2: THE CRITICAL ATTEMPTED CRIME RATE  $\hat{\alpha}_\xi$  INCREASES IN THE INDEX  $\xi$ .** By definition,  $\hat{\alpha}_\xi \equiv \operatorname{argmax}_\alpha \bar{\kappa}(\alpha, \xi)$ . By Topkis' Theorem (Topkis 1998),  $\hat{\alpha}_\xi$  rises in  $\xi$  if  $\bar{\kappa}_{\alpha\xi}(\alpha, \xi) > 0$ . I argue that this is positive, i.e., that  $\bar{\kappa}_{\alpha\xi} = -\hat{\phi}_\xi - \alpha \hat{\phi}_{\alpha\xi} > 0$ .

Log-differentiate the  $\alpha$  derivative  $M = V_{\phi\phi} \hat{\phi}_\alpha$  of the FOC (3) in  $\xi$  to get

$$\begin{aligned} (A3) \quad 0 &= \frac{V_{\phi\phi\phi} \hat{\phi}_\xi}{V_{\phi\phi}} + \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} + \frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} \Rightarrow \frac{-\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} \\ &= -\frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \cdot \frac{V_{\phi\xi}}{V_{\phi\phi}} + \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} = \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right), \end{aligned}$$

where  $\hat{\phi}_\xi = -V_{\phi\xi} / V_{\phi\phi}$ , found by differentiating (3). Since  $\alpha \hat{\phi}_\alpha = V_\phi / V_{\phi\phi}$  by (3),  $\bar{\kappa}_{\alpha\xi} > 0$  if and only if

$$\frac{-\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} > \frac{\hat{\phi}_\xi}{\alpha \hat{\phi}_\alpha} \Leftrightarrow \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right) > \frac{-V_{\phi\xi} V_{\phi\phi}}{V_{\phi\phi} V_\phi} \Leftrightarrow \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} > \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_\phi}.$$

This last inequality holds, because the right side is nonpositive, as  $V_\phi$  is log-concave in  $\phi$ ; and  $V_{\phi\phi\xi}, V_{\phi\xi} > 0$ , as  $V_\phi$  and  $V$  are supermodular and log-supermodular in  $(\phi, \xi)$ , respectively. ■

Next, I show that the aggregate crime rate function  $\tilde{\kappa}(\alpha) \equiv \int_0^1 \bar{\kappa}(\alpha, \xi) dG(\xi)$  is quasi-concave in  $\alpha$ . Building on the variation diminishing property of totally positive functions (Karlin 1968), Choi and Smith (2017) provide a useful condition that ensures that the weighted sum of quasi-concave functions is quasi-concave.

To this end, decompose the individual crime rate function  $\bar{\kappa} = \bar{\kappa}^I + \bar{\kappa}^D$  into its increasing and decreasing portions,  $\bar{\kappa}^I$  and  $\bar{\kappa}^D$ , respectively, where  $\bar{\kappa}^I$  and  $-\bar{\kappa}^D$  are monotone, and  $\bar{\kappa}^I$  (respectively,  $-\bar{\kappa}^D$ ) is constant right (respectively, left) of the peak (argmax) of  $\bar{\kappa}$ . When these functions are differentiable, for any  $\xi, \xi'$ , say that  $-\bar{\kappa}^D(\cdot, \xi)$  grows proportionally faster than  $\bar{\kappa}^I(\cdot, \xi')$ , if  $\bar{\kappa}^I(\cdot, \xi')$  is more risk averse than  $-\bar{\kappa}^D(\cdot, \xi)$ , namely

$$(A4) \quad -\bar{\kappa}_{\alpha\alpha}^I(\alpha, \xi') / \bar{\kappa}_{\alpha}^I(\alpha, \xi') \geq -\bar{\kappa}_{\alpha\alpha}^D(\alpha, \xi) / \bar{\kappa}_{\alpha}^D(\alpha, \xi).$$

More generally, for any  $\alpha_3 \geq \alpha_2 \geq \alpha_1$ ,

$$(A5) \quad [\bar{\kappa}^I(\alpha_2, \xi') - \bar{\kappa}^I(\alpha_1, \xi')] [\bar{\kappa}^D(\alpha_2, \xi) - \bar{\kappa}^D(\alpha_3, \xi)] \\ \geq [\bar{\kappa}^I(\alpha_3, \xi') - \bar{\kappa}^I(\alpha_2, \xi')] [\bar{\kappa}^D(\alpha_1, \xi) - \bar{\kappa}^D(\alpha_2, \xi)].$$

By proposition 1 in Choi and Smith (2017), the crime rate  $\tilde{\kappa}$  is quasi-concave in  $\alpha$  if and only if  $-\bar{\kappa}^D(\cdot, \xi)$  grows proportionally faster than  $\bar{\kappa}^I(\cdot, \xi')$ , for all  $\xi', \xi$ .

**Step 3:**  $-\bar{\kappa}^D(\cdot, \xi)$  GROWS PROPORTIONALLY FASTER THAN  $\bar{\kappa}^I(\cdot, \xi')$ , FOR ALL  $\xi, \xi'$ . Whenever  $\bar{\kappa}^D(\cdot, \xi)$  or  $\bar{\kappa}^I(\cdot, \xi)$  is constant, (A5) holds, since both sides vanish.

Assume first  $\xi' \leq \xi$ . Then the claim holds—for if  $\alpha < \hat{\alpha}_{\xi}$ , then  $-\bar{\kappa}^D(\alpha, \xi)$  is constant, and if  $\alpha \geq \hat{\alpha}_{\xi}$ , then  $\bar{\kappa}^I(\alpha, \xi')$  is constant (recalling that  $\hat{\alpha}_{\xi} \geq \hat{\alpha}_{\xi'}$ , by Step 2).

The case  $\xi' > \xi$  is trickier, as both  $\bar{\kappa}^I(\alpha, \xi')$  and  $-\bar{\kappa}^D(\alpha, \xi')$  are increasing on an interval, namely, for  $\alpha \in [\hat{\alpha}_{\xi}, \min\{\bar{\alpha}_{\xi}, \hat{\alpha}_{\xi'}\}]$ . Here,  $-\bar{\kappa}^D(\cdot, \xi)$  and  $\bar{\kappa}^I(\cdot, \xi')$  are differentiable, and so I can use criterion (A4). But on this interval,  $\bar{\kappa}^I(\alpha, \xi) = \bar{\kappa}(\alpha, \xi)$  is increasing and concave, whereas  $\bar{\kappa}^D(\alpha, \xi') = \bar{\kappa}(\alpha, \xi')$  is decreasing and concave since  $\bar{\kappa}(\cdot, \xi)$  and  $\bar{\kappa}(\cdot, \xi')$  are concave, by Step 1. As a result,  $\bar{\kappa}^I(\cdot, \xi')$  is more risk averse than  $-\bar{\kappa}^D(\cdot, \xi)$ :

$$-\bar{\kappa}_{\alpha\alpha}^I(\alpha, \xi') / \bar{\kappa}_{\alpha}^I(\alpha, \xi') > 0 > -\bar{\kappa}_{\alpha\alpha}^D(\alpha, \xi) / \bar{\kappa}_{\alpha}^D(\alpha, \xi). \quad \blacksquare$$

**PROOF OF PROPOSITION 1:**

Suppose that punishment  $\ell$  rises. As depicted in Figure 2, as  $\ell$  rises the supply curve  $\mathcal{S}$  shifts left, while the demand curve  $\mathcal{D}$  holds constant. The attempted crime rate  $\alpha^*$  and failure rate  $\varphi^*$  unambiguously fall: fewer crimes are attempted, but each succeeds more often, as potential victims grow less vigilant. Since supply  $\mathcal{S}$  shifts

along demand  $\mathcal{D}$ , the crime rate can be written as  $\kappa^*(\ell) \equiv \alpha^*(\ell)(1 - \mathcal{D}(\alpha^*(\ell)))$ . Observe that when penalties  $\ell = M$ , no criminal obtains revenues; thus, there are no attempted crimes  $\alpha^*(M) = 0$ , and so  $\kappa^*(M) = 0$ . But when legal penalties  $\ell = 0$ , the attempted crime rate is clearly positive and below the attempted crime rate ceiling, i.e.,  $\alpha^*(0) \in (0, \bar{\alpha})$ , whereas the failure rate is imperfect  $\varphi^*(0) < 1$ . Thus, since  $\alpha^*(\ell) \in [0, \alpha^*(0)]$  is monotone decreasing, and the map  $\alpha \mapsto \alpha(1 - \mathcal{D}(\alpha))$  is quasi-concave (Lemma A.1), it follows that  $\kappa^*(\ell)$  is quasi-concave in  $\ell \in [0, M]$ . Finally, if the demand is sufficiently elastic at  $\alpha = \alpha^*(0)$ , then the map  $\alpha \mapsto \alpha(1 - \mathcal{D}(\alpha))$  is decreasing around  $\alpha = \alpha^*(0)$ , and so the crime rate  $\kappa^*(\ell)$  is increasing around  $\alpha = \alpha^*(0)$  because  $\kappa^*(\cdot)$  is the composition of two decreasing functions. ■

#### A6. Higher Punishment Increases Crime: Proof of Proposition 2

Recall that the crime rate falls along the demand curve if the demand is sufficiently elastic (Proposition 1). Equivalently,  $(1 - \mathcal{D}(\alpha))/(\alpha\mathcal{D}'(\alpha)) < 1$ . This condition can be rewritten as  $\kappa M/(\alpha^2 M\mathcal{D}'(\alpha)) < 1$ , where  $\kappa = (1 - \mathcal{D}(\alpha))\alpha$  is the crime rate. Now, differentiate the demand curve  $\mathcal{D}(\alpha)$  in (4) to get

$$\begin{aligned} \alpha^2 M\mathcal{D}'(\alpha) &= \alpha M \int_0^1 \frac{V_\phi(\hat{\phi}|\xi)}{V_{\phi\phi}(\hat{\phi}|\xi)} dG(\xi) \geq \int_0^1 V_\phi(\hat{\phi}|\xi) \frac{V_\phi(\hat{\phi}|\xi)}{V_{\phi\phi}(\hat{\phi}|\xi)} dG(\xi) \\ &= \int_0^1 V(\hat{\phi}|\xi) \frac{V_\phi(\hat{\phi}|\xi)/V(\hat{\phi}|\xi)}{V_{\phi\phi}(\hat{\phi}|\xi)/V_\phi(\hat{\phi}|\xi)} dG(\xi) \geq \int_0^1 V(\hat{\phi}|\xi) dG(\xi). \end{aligned}$$

To get the first equality, log-differentiate the FOC (3). To get the first inequality, use that optimal  $\hat{\phi}$  obeys  $\alpha M \geq V_\phi(\hat{\phi}(\alpha|\xi)|\xi)$  for all types  $\xi$ . Finally, use that  $(V_\phi/V)/(V_{\phi\phi}/V_\phi) \geq 1$ , as it is well-known that if  $\log V_\phi$  is concave in  $\phi$ , so is  $\log V$  by Prékopa's Theorem (Prékopa 1973). Essentially, I used the fact that log-concavity is passed from functions to their integrals; see, e.g., lemma 3 in Bagnoli and Bergstrom (2005). Altogether, if  $\kappa M < \int_0^1 V(\hat{\phi}|\xi) dG(\xi)$ , then  $\kappa M/(\alpha^2 M\mathcal{D}'(\alpha)) < 1$ , and thus the crime rate falls along the demand, or the criminal Laffer curve is upward sloping. ■

#### A7. Individual Crime and Punishment: Proof of Proposition 3

First, write the *individual crime rate function* for victim  $\xi$  as  $\bar{\kappa}(\alpha, \xi) \equiv \alpha(1 - \hat{\phi}(\alpha, \xi))$ . Next, let  $\alpha^*(\ell)$  be the equilibrium attempted crime rate for punishment  $\ell \in [0, M]$ . Now differentiate the individual crime rate function in  $\ell$  to get  $d\bar{\kappa}/d\ell = \bar{\kappa}_\alpha(\alpha^*(\ell), \xi) \cdot (d\alpha^*(\ell)/d\ell)$ . Since  $d\alpha^*(\ell)/d\ell < 0$  by Proposition 1, the individual crime rate for victim  $\xi$  increases when punishment  $\ell$  increases if and only if  $\bar{\kappa}_\alpha(\alpha^*(\ell), \xi) < 0$ . Also, by the Proof of Proposition 1 (see Step 2),  $\bar{\kappa}(\alpha, \xi)$  is supermodular in  $(\alpha, \xi)$ , and thus  $\bar{\kappa}_\alpha(\alpha^*(\ell), \xi)$  is increasing in  $\xi$ . Thus, there exists a critical type  $\bar{\xi}$  such that  $\bar{\xi}$  equals one if  $\bar{\kappa}_\alpha(\alpha^*(\ell), 1) < 0$ , equals zero if  $\bar{\kappa}_\alpha(\alpha^*(\ell), 0) > 0$ , and solves  $\bar{\kappa}_\alpha(\alpha^*(\ell), \bar{\xi}) = 0$  otherwise. Therefore, a marginal increase in punishment  $\ell$  raises the crime rate for potential victim  $\xi$  if

and only if  $\xi \leq \bar{\xi}$ . Finally,  $\bar{\xi}$  falls in  $\ell$  since  $\bar{\kappa}(\alpha, \xi)$  is also quasi-concave in  $\alpha$  (Appendix A.A5). ■

A8. The Role of Vigilance Costs: Proof of Proposition 4

The proof of part (i) is given in the main text. I now prove part (ii).

LEMMA A.2: Along the demand  $\mathcal{D}$ , the average success rate  $1 - \mathcal{D}$  is more elastic with  $G_H$  than with  $G_L$  if the likelihood ratio  $g_H(\xi)/g_L(\xi)$  is increasing.

PROOF:

As in Section A.A2, consider a parameterized type-distribution  $H : [0, 1] \times \{0, 1\} \rightarrow [0, 1]$  with  $(\xi, \theta) \mapsto H$  and density  $h(\xi, \theta)$ . Also, let  $H(\xi, 0) \equiv G_L(\xi)$  and  $H(\xi, 1) \equiv G_H(\xi)$ . This implies that  $h(\xi, \theta)$  is log-supermodular in  $(\xi, \theta)$ ; see Step 1 in Section A.A2.

**Step 1:**  $\hat{\phi}_\alpha(\alpha|\xi)/(1 - \hat{\phi}(\alpha|\xi))$  RISES IN  $\xi$ . Log-differentiate this expression in  $\xi$  to get

$$\frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} + \frac{\hat{\phi}_\xi}{1 - \hat{\phi}} = \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{1}{1 - \hat{\phi}} \right),$$

where I used (A1) and  $\hat{\phi}_\xi = -V_{\phi\xi}/V_{\phi\phi}$ , which was found in Section A.A2 (Step 2). Next, I show that the (displayed) expression above is negative. Indeed, adding and subtracting  $V_{\phi\phi}/V_\phi$  to its right-hand side,

$$\begin{aligned} & \frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{1}{1 - \hat{\phi}} \right) \\ &= \frac{V_{\phi\xi}}{V_{\phi\phi}} \left[ \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_\phi} \right) + \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} \right) - \frac{1}{1 - \hat{\phi}} \right] < 0. \end{aligned}$$

The inequality holds, for  $\log(V_\phi)$  is concave in  $\phi$  and supermodular in  $(\phi, \xi)$ ; also,  $V_{\phi\xi}, V_{\phi\phi} \geq 0$ .

**Step 2:**  $1 - \mathcal{D}$  IS LOG-SUPERMODULAR IN  $(\xi, \theta)$ . By lemma 4 in Athey (2002), since  $h(\xi, \theta)$  is log-supermodular in  $(\xi, \theta)$ , we have that

$$\frac{\int (1 - \hat{\phi}(\alpha|\xi))h(\xi, \theta) d\xi}{\int \hat{\phi}_\alpha(\alpha|\xi)h(\xi, \theta) d\xi} = \frac{1 - \mathcal{D}(\alpha, \theta)}{\mathcal{D}_\alpha(\alpha, \theta)}$$

is increasing in  $\theta$ . Thus,  $\mathcal{E}_\alpha(1 - \mathcal{D}) = -\alpha \mathcal{D}_\alpha / (1 - \mathcal{D})$  is increasing in  $\theta$ . ■

PROOF OF (ii):

The criminal Laffer curve shifts up at every punishment  $\ell$  (left panel of Figure 3). Now I show that it also shifts left. First, call  $\ell_{peak} \geq 0$  the punishment  $\ell$  for which

the crime rate  $\kappa^*(\ell)$  peaks. Notice that  $\ell_{peak}$  uniquely solves  $\alpha^*(\ell_{peak}) = \alpha_{peak}$ , where  $\alpha_{peak}$  maximizes  $\alpha(1 - \mathcal{D}(\alpha))$ . Next, recall that optimizers are invariant to monotonic transformations of the objective function. So,  $\alpha_{peak} = \operatorname{argmax}_{\alpha} \log(\alpha) + \log(1 - \mathcal{D})$ . But  $\partial \log(1 - \mathcal{D}) / \partial \alpha$  rises as  $G$  rises, by Lemma A.2; so  $\alpha_{peak}$  rises as  $G$  rises, by Topkis' Theorem (Topkis 1998). Finally, since  $\alpha^*$  falls in  $\ell$  (Proposition 1),  $\ell_{peak}$  must fall as  $G$  rises. ■

A9. *The Stakes of Crime: Proof of Proposition 5*

Parameterize the demand  $\mathcal{D}$  (and individual demands  $\hat{\phi}$ ) and supply  $\mathcal{S}$  curves so that  $\mathcal{D}(\alpha, M) \equiv \int \hat{\phi}(\alpha, \xi, M) dG$  and  $\mathcal{S}(\varphi, M) \equiv F((1 - \varphi)M - \ell)$ , respectively. The following observations are useful. (★): (i)  $\mathcal{S}$  vanishes, i.e.  $\mathcal{S} \equiv 0$ , for all  $\varphi$  when stakes are low, i.e., when  $M \leq \ell$ . (ii) Second,  $M\hat{\phi}_M = V_{\phi} / V_{\phi\phi}$  by log-differentiating the FOC (3) in  $M$ .

CLAIM A.2: *The equilibrium failure rate  $\varphi^*$  rises as  $M$  rises.*

PROOF:

First, as  $M$  rises potential victims raise their vigilance, i.e.,  $\hat{\phi}_M \geq 0$ . On the other side, criminals attempt more crime; i.e., the marginal criminal  $\bar{c}_M > 0$ . As seen in Figure 4, both the demand  $\mathcal{D}$  and supply  $\mathcal{S}$  shift up; thus, the failure rate  $\varphi^*$  unambiguously rises. ■

As discussed in the main text, the attempted crime rate  $\alpha^*$  rises if and only if supply  $\mathcal{S}$  shifts up more than demand  $\mathcal{D}$  does. To get the respective magnitudes of the vertical demand shift  $\varphi_M^D$  and the vertical supply shift  $\varphi_M^S$ , fix  $\alpha = \alpha^*$  and, respectively, differentiate in  $M$  the equilibrium conditions,  $\mathcal{D}(\alpha^*, M) = \varphi^*$  and  $\mathcal{S}(\varphi^*, M) = \alpha^*$ . After few algebraic manipulations,

$$(A6) \quad \varphi_M^D \equiv \int \hat{\phi}_M(\alpha^*, \xi, M) dG = \int \frac{V_{\phi}}{MV_{\phi\phi}} dG,$$

and

$$\varphi_M^S \equiv \frac{1 - \varphi^*}{M} = \int \frac{1 - \hat{\phi}(\alpha^*, \xi, M)}{M} dG.$$

CLAIM A.3: *If demand  $\mathcal{D}$  is sufficiently elastic for some  $M$ , then the equilibrium attempted crime rate  $\alpha^*$  is hump shaped in the stakes. Otherwise,  $\alpha^*$  is monotone increasing.*

PROOF:

**Step 1:** THE ATTEMPTED CRIME RATE RISES FOR SMALL STAKES. Consider the shift-difference  $\varphi_M^D - \varphi_M^S$  and small stakes  $0 < M \leq \ell$ . By observation (★)–(i), the supply  $\mathcal{S} \equiv 0$  for all  $\varphi$ , and so the supply and demand curves intersect at the

origin:  $\varphi^* = \alpha^* = 0$ . So  $\varphi_M^S = 1/M$ , while  $\varphi_M^D < 1/M$  since  $\hat{\phi}(0, \xi, M) = 0$  and  $\lim_{\phi \downarrow 0} V_{\phi\phi}/V_\phi > 1$  (see Section II).

**Step 2:** THE ATTEMPTED CRIME RATE FALLS IF AND ONLY IF DEMAND IS SUFFICIENTLY ELASTIC. By (A6), the shift-difference  $\varphi_M^D - \varphi_M^S \geq 0$  if and only if  $\int (V_\phi/V_{\phi\phi}) dG \geq 1 - \varphi^*$ . But this latter inequality is equivalent to  $\mathcal{D}$  being sufficiently elastic. Indeed, notice that, in equilibrium,  $1 - \mathcal{D} = 1 - \varphi^*$ , and  $\alpha^* \mathcal{D}_\alpha = \int \alpha^* \hat{\phi}_\alpha dG = \int (V_\phi/V_{\phi\phi}) dG$ , by log-differentiating the FOC (3) in  $\alpha$ . Thus,  $\varphi_M^D \geq \varphi_M^S \Leftrightarrow \int (V_\phi/V_{\phi\phi}) dG \geq 1 - \varphi^* \Leftrightarrow \mathcal{E}_\alpha(\mathcal{D}) \geq (1 - \mathcal{D})/\mathcal{D}$ .

**Step 3:** THE SHIFT-DIFFERENCE IS MONOTONE IN  $M$ . First, assume that demand  $\mathcal{D}$  is sufficiently elastic for some high stakes  $M$ . Then, the shift-difference is positive for high  $M$  and negative for small  $M$ , by Steps 1 and 2. Next, I'll show that the shift-difference is monotone, ensuring a single-crossing. Differentiate the shift-difference in  $M$  to get

$$\begin{aligned} \frac{\partial(\varphi_M^D - \varphi_M^S)}{\partial M} &= \int \left( \frac{V_\phi}{M V_{\phi\phi}} \right) \left( \frac{V_{\phi\phi} \hat{\phi}_M}{V_\phi} - \frac{1}{M} - \frac{V_{\phi\phi\phi} \hat{\phi}_M}{V_{\phi\phi}} \right) + \left( \frac{\hat{\phi}_M}{M} + \frac{1 - \hat{\phi}}{M^2} \right) dG \\ &= \int \left[ \left( \frac{\hat{\phi}_M V_\phi}{M V_{\phi\phi}} \right) \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{1}{M \hat{\phi}_M} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} + \frac{V_{\phi\phi}}{V_\phi} \right) + \frac{1 - \hat{\phi}}{M^2} \right] dG \\ &= \int \left[ - \left( \frac{\hat{\phi}_M V_\phi}{M V_{\phi\phi}} \right) \left( \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} - \frac{V_{\phi\phi}}{V_\phi} \right) + \frac{1 - \hat{\phi}}{M^2} \right] dG > 0, \end{aligned}$$

where I used observation (★)–(ii) for the last equality. Now, the inequality holds since  $\hat{\phi}, V_\phi, V_{\phi\phi} \geq 0$ , and  $V_\phi$  is log-concave in  $\phi$  (and so  $V_{\phi\phi\phi}/V_{\phi\phi} \leq V_{\phi\phi}/V_\phi$ ) and  $\hat{\phi} < 1$ .

Finally, because the shift-difference is negative for small stakes (Step 1), is positive for high stakes (Step 2), and the shift-difference is monotone (Step 3), there exists a unique intermediate stake such that the attempted crime rate rises if and only if stakes are below this critical value. In other words, the attempted crime rate  $\alpha^*$  is hump shaped in  $M$ . ■

CLAIM A.4: *If demand  $\mathcal{D}$  is sufficiently elastic for some stake  $M$ , then the equilibrium crime rate  $\kappa^*$  initially rises and eventually falls as the stake rises.*

PROOF:

First, recall that if  $\mathcal{D}$  is sufficiently elastic for high  $M$ , then  $\alpha^*$  is decreasing in  $M$  by Claim A.2. Next, since the failure rate  $\varphi^*$  is increasing in  $M$  (by Claim A.1), the success rate  $1 - \varphi^*$  is decreasing in  $M$ . So, for high enough stakes, the crime rate  $\kappa^* = \alpha^*(1 - \varphi^*)$  falls in  $M$  since it is the product of two decreasing functions. Now, for small stakes,  $\alpha^* = \varphi^* = 0$ , and so  $\kappa^* = 0$ . Finally, since  $\kappa^*$  is strictly positive otherwise,  $\kappa^*$  must initially rise. ■

CLAIM A.5: *Potential victims with low cost  $\xi$  respond with more vigilance than those with high cost, as stakes  $M$  rises.*



PROOF:

Call  $\alpha^*(M)$  the equilibrium attempted crime rate for stakes  $M$  and  $\hat{\phi}(\alpha^*(M), M|\xi)$  the optimal failure rate for type  $\xi$  when the stake is  $M$  and the attempted crime rate  $\alpha^*(M)$ .

**Step 1:** THE INDIVIDUAL FAILURE RATE  $\hat{\phi}(\alpha^*(M), M|\xi)$  RISES IN  $M$ . To see this, differentiate  $\hat{\phi}(\alpha^*(M), M|\xi)$  in  $M$  to get

$$(A7) \quad \frac{d\hat{\phi}}{dM} = \hat{\phi}_\alpha(\alpha^*(M), M|\xi) \frac{d\alpha^*}{dM} + \hat{\phi}_M(\alpha^*(M), M|\xi).$$

Next, log-differentiate the FOC (3) in  $\alpha$  and in  $M$  to respectively get  $\hat{\phi}_\alpha = V_\phi / (\alpha V_{\phi\phi}) > 0$  and  $\hat{\phi}_M = V_\phi / (MV_{\phi\phi}) > 0$ . Thus,  $\hat{\phi}_M = (\alpha/M) \hat{\phi}_\alpha$ , and so (A7) can be written as

$$\frac{d\hat{\phi}}{dM} = \hat{\phi}_\alpha(\alpha^*(M), M|\xi) \alpha^*(M) \left( \frac{d\alpha^*}{dM} \cdot \frac{1}{\alpha} + \frac{1}{M} \right).$$

I now argue that  $\left( \frac{d\alpha^*}{dM} \cdot \frac{1}{\alpha} + \frac{1}{M} \right) > 0$  and so  $\frac{d\hat{\phi}}{dM} > 0$  since  $\hat{\phi}_\alpha > 0$ . Indeed, by Claim A.2,  $\varphi^*(M)$  rises in  $M$ ; i.e.,

$$\frac{d\varphi^*(M)}{dM} = \int \frac{d\hat{\phi}}{dM} dG(\xi) > 0.$$

This implies that  $\left( \frac{d\alpha^*}{dM} \cdot \frac{1}{\alpha} + \frac{1}{M} \right) > 0$ . If not, then  $\frac{d\hat{\phi}}{dM} \leq 0$  for all types  $\xi$  since  $\hat{\phi}_\alpha > 0$ . But then  $\frac{d\varphi^*(M)}{dM} = \int \frac{d\hat{\phi}}{dM} dG(\xi) \leq 0$ , a contradiction.

**Step 2:**  $\hat{\phi}_\alpha(\alpha, M|\xi)$  IS DECREASING IN  $\xi$ . To see this, use expression (A3) to get

$$\frac{\hat{\phi}_{\alpha\xi}}{\hat{\phi}_\alpha} = \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \cdot \frac{V_{\phi\xi}}{V_{\phi\phi}} - \frac{V_{\phi\phi\xi}}{V_{\phi\phi}} = -\frac{V_{\phi\xi}}{V_{\phi\phi}} \left( \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right).$$

Next, I show that  $\left( \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right) > 0$  and so  $\hat{\phi}_{\alpha\xi} < 0$  since  $\hat{\phi}_\alpha > 0$  and  $V_{\phi\xi}, V_{\phi\phi} > 0$  by assumption. Indeed, add and subtract  $\frac{V_{\phi\phi\xi}}{V_\phi}$  to  $\left( \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right)$  to get

$$\frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} = \left( \frac{V_{\phi\phi\xi}}{V_{\phi\xi}} - \frac{V_{\phi\phi}}{V_\phi} \right) + \left( \frac{V_{\phi\phi}}{V_\phi} - \frac{V_{\phi\phi\phi}}{V_{\phi\phi}} \right).$$

Now, notice that the left parenthesized term is positive because  $V_\phi$  is log-supermodular in  $(\phi, \xi)$ , whereas the right parenthesized one is also positive because  $V_\phi$  is log-concave in  $\phi$ .

**Step 3:**  $\frac{d^2\hat{\phi}}{dMd\xi} < 0$ . By Steps 1 and 2,

$$\frac{d^2\hat{\phi}}{dMd\xi} = \underbrace{\hat{\phi}_{\alpha\xi}(\alpha^*(M), M|\xi)}_{<0} \alpha^*(M) \underbrace{\left( \frac{d\alpha^*}{dM} \cdot \frac{1}{\alpha} + \frac{1}{M} \right)}_{>0} < 0.$$

In other words, after an increase in the stakes of crime  $M$ , high-cost individuals raise their vigilance less than low-cost ones. ■

A10. *Depicting Potential Victims' Welfare Losses*

Use the FOC (3) and that  $\hat{\phi}(0|\xi) = 0 = V_\phi(0|\xi)$  to get

$$\begin{aligned} v^* &= \int_0^1 V(\hat{\phi}(\alpha^*|\xi)|\xi) dG(\xi) = \int_0^1 \left( \int_0^{\alpha^*} V_\phi(\hat{\phi}(\alpha|\xi)|\xi) \hat{\phi}_\alpha(\alpha|\xi) d\alpha \right) dG(\xi) \\ &= \int_0^1 \int_0^{\alpha^*} \alpha M \hat{\phi}_\alpha(\alpha|\xi) d\alpha dG(\xi). \end{aligned}$$

Now integrate by parts to get  $\int_0^{\alpha^*} \alpha \hat{\phi}_\alpha(\alpha|\xi) d\alpha = \alpha^* \hat{\phi}(\alpha^*|\xi) - \int_0^{\alpha^*} \hat{\phi}(\alpha|\xi) d\alpha$ . Therefore,

$$\begin{aligned} v^*/M &= \int_0^1 \int_0^{\alpha^*} \alpha \hat{\phi}_\alpha(\alpha|\xi) d\alpha dG(\xi) \\ &= \int_0^1 \alpha^* \hat{\phi}(\alpha^*|\xi) dG(\xi) - \int_0^1 \int_0^{\alpha^*} \hat{\phi}(\alpha|\xi) d\alpha dG(\xi) \\ &= \int_0^1 \alpha^* \hat{\phi}(\alpha^*|\xi) dG(\xi) - \int_0^{\alpha^*} \int_0^1 \hat{\phi}(\alpha|\xi) dG(\xi) d\alpha \\ &= \alpha^* \varphi^* - \int_0^{\alpha^*} \mathcal{D}(\alpha) d\alpha = \int_0^{\alpha^*} [\varphi^* - \mathcal{D}(\alpha)] d\alpha \\ &= \int_0^{\alpha^* M} [\varphi^* - \mathcal{D}(\tilde{\alpha}/M)] d\tilde{\alpha}/M, \end{aligned}$$

where I have changed variables so that  $\tilde{\alpha} = \alpha M$  and used Fubini's Theorem to exchange the double integral. Altogether,  $v^* = \int_0^{\alpha^* M} [\varphi^* - \mathcal{D}(\alpha)] d(\alpha M)$ , namely, *vigilance expenditures  $v^*$  equals the area below equilibrium failure rate  $\varphi^*$  and above the demand  $\mathcal{D}(\alpha)$ .* ■

A11. *The Victim-Optimal Levels of Crime and Vigilance*

First, notice that the optimal failure rate  $\varphi^o$  must be positive since raising  $\varphi$  from zero is costless at the margin, as  $V_\phi(0|\xi) = 0$  for all  $\xi$  and  $\mathcal{MS}(0) < 0$ . Also,  $\varphi^o < 1$  since a marginal drop of  $\varphi$  from  $\varphi = 1$  has no marginal impact on the crime rate, i.e.,  $\mathcal{MS}(1) = 0$ . Thus, the optimal failure rate is positive and imperfect, i.e.,  $\varphi^o \in (0, 1)$ , and so it solves the FOC

$$\mathcal{S}(\varphi^o)M - \mathcal{S}'(\varphi^o)(1 - \varphi^o)M = \int_0^1 V_\phi(\hat{\phi}^o(\varphi^o|\xi)|\xi) \hat{\phi}_\varphi^o(\varphi^o|\xi) dG(\xi).$$

Next, by the definition of  $\hat{\phi}^o$ ,  $\int_0^1 \hat{\phi}^o(\varphi|\xi) dG(\xi) = \varphi$ , and so  $\int_0^1 \hat{\phi}_\varphi^o(\varphi|\xi) dG(\xi) = 1$ . Also,  $V_\phi(\hat{\phi}^o(\varphi|\xi)|\xi) = \mathcal{D}^{-1}(\varphi)M$ , again by construction of  $\hat{\phi}^o$ . Thus, the FOC above simplifies to (8). The attempted crime rate is the mass of potential criminals that attempt a crime when  $\varphi = \varphi^o$ , namely,  $\mathcal{S}(\varphi^o)$ , and the individual failure rates obey  $\hat{\phi}^o(\varphi^o|\xi)$  for all  $\xi$ .

Finally, since  $\mathcal{E}_{1-\varphi}(f) \geq -2$ , the crime rate  $\mathcal{S}(\varphi)(1 - \varphi)$  is convex in  $\varphi$ , and so is the program (7), as  $\mathcal{D}^{-1}(\varphi)$  is increasing in  $\varphi$ . So  $(\varphi^o, \alpha^o, \hat{\phi}^o)$  solves the planner's problem. ■

A12. *The Inefficiency Gap: Proof of Proposition 6 Finished*

*Optimal Vigilance.*—Differentiate the objective function (7) in  $\ell$ , using (5), to get  $\mathcal{S}_\ell(\varphi)(1 - \varphi)M = -f[(1 - \varphi)M - \ell](1 - \varphi)M$ . Next, differentiate this latter expression in  $\varphi$  to obtain

$$\begin{aligned} & f'[(1 - \varphi)M - \ell](1 - \varphi)M^2 + f[(1 - \varphi)M - \ell]M \\ &= f(\bar{c}(\varphi))(\mathcal{E}_{1-\varphi}(f) + 1)M. \end{aligned}$$

So the objective function is submodular in  $(\varphi, \ell)$  if and only if  $\mathcal{E}_{1-\varphi}(f(\bar{c})) \leq -1$ . Thus, by Topkis (1998), the minimizer  $\varphi^o$  rises as  $\ell$  rises if and only if  $\mathcal{E}_{1-\varphi}(f(\bar{c})) \leq -1$ .

Next, notice that (i) if  $[(c + \ell)f(c)]' \leq 0$ , then  $(c + \ell)f'(c)/f(c) \leq -1$ , and so at  $c = \bar{c}$

$$\mathcal{E}_{1-\varphi}(f(\bar{c})) = \frac{f'[(1 - \varphi)M - \ell](1 - \varphi)M}{f[(1 - \varphi)M]} = \frac{f'(\bar{c})(\bar{c} + \ell)}{f(\bar{c})} \leq -1,$$

where I used the definition of  $\bar{c}(\varphi)$  below (2). Thus, the minimizer  $\varphi^o$  rises as  $\ell$  rises. Finally, following the same logic, (ii) if  $[(c + \ell)f(c)]' \geq 0$ , then  $(c + \ell)f'(c)/f(c) \geq -1$ , and so at  $c = \bar{c}$

$$\mathcal{E}_{1-\varphi}(f(\bar{c})) = \frac{f'[(1 - \varphi)M - \ell](1 - \varphi)M}{f[(1 - \varphi)M]} = \frac{f'(\bar{c})(\bar{c} + \ell)}{f(\bar{c})} \geq -1.$$

Therefore,  $\varphi^o$  falls as  $\ell$  rises, by the previous arguments.

*Inefficiency Gap.*—Now, I address the claims regarding the inefficiency gap. Part (i) is proved in the main text.

Part (ii): Differentiate the right side of (9) to get

$$\frac{\partial \mathcal{E}_{1-\varphi}(\mathcal{S})}{\partial \ell} = \underbrace{\frac{\partial}{\partial \varphi} \left( \frac{f(1 - \varphi^o)M}{F} \right) \frac{\partial \varphi^o}{\partial \ell}}_{(\clubsuit)} + \underbrace{\frac{\partial}{\partial \ell} \left( \frac{f(1 - \varphi^o)M}{F} \right)}_{(\spadesuit)}.$$

Next, I note that  $(\clubsuit)$  is positive. Indeed, after some algebra,

$$\frac{\partial}{\partial \varphi} \left( \frac{f(1 - \varphi^o)M}{F} \right) = -\mathcal{E}_{1-\varphi}(\mathcal{S})M \left( \frac{f'}{f} - \frac{f}{F} + \frac{1}{(1 - \varphi^o)M} \right) < 0,$$

where the inequality follows as  $-\mathcal{E}_{1-\varphi}(\mathcal{S})M < 0$  and

$$\frac{f'}{f} - \frac{f}{F} + \frac{1}{(1 - \varphi^o)M} > \frac{f'}{f} - \frac{f}{F} + \frac{1}{(1 - \varphi^o)M - \ell} \geq 0.$$

The last inequality holds since  $(f/F)c$  is monotone in  $c$ . Thus,  $(\clubsuit) > 0$  since the  $\partial\varphi^o/\partial\ell < 0$  as previously argued. Next,  $(\spadesuit)$  is positive since  $(f/F)$  is decreasing in  $c$  and so increasing in  $\ell$ . Altogether, the relative excess of demand rises as punishment rises, given (9). ■

APPENDIX B. MINIMIZING THE SOCIAL COSTS OF CRIME

Suppose the planner’s goal is to minimize the social costs of crime (6). The planner can minimize social costs by choosing an average failure rate  $\varphi^{sc} \in [0, 1]$ , individual demands  $\hat{\phi}^o(\varphi^{sc}|\xi)$ , and an attempted crime rate  $\alpha^{sc} = \mathcal{S}(\varphi^{sc})$ , where  $\varphi^{sc}$  solves

$$(B1) \quad \min_{\varphi \in [0,1]} \mathcal{S}(\varphi)(1 - \varphi)M + \int_0^1 V(\hat{\phi}^o(\varphi|\xi)|\xi) dG(\xi) - \int_0^{(1-\varphi)M-\ell} F(c) dc.$$

As argued in Section VI, the last term in (B1) equals total criminal profits. To have a convex program, I assume that the density elasticity  $\mathcal{E}_{1-\varphi}(f) \geq -1$  for  $c = \bar{c}(\varphi)$ . This condition holds if the cost distribution  $F$  is either convex or not too concave.

Next, notice that the average failure rate  $\varphi^{sc}$  must be interior. Indeed, differentiate the objective function (B1) in  $\varphi$  and evaluate it at  $\varphi = 0$  and  $\varphi = 1$ . In the former case, the marginal returns of raising  $\varphi$  from  $\varphi = 0$  are negative and equal to  $\mathcal{S}'(0)M < 0$ . In the latter case, the absolute marginal returns of lowering  $\varphi$  from  $\varphi = 1$  are positive and equal to  $\mathcal{D}^{-1}(1) > 0$ . Thus,  $\varphi^{sc} \in (0, 1)$  and is characterized by the FOC

$$(B2) \quad \mathcal{S}(\varphi^{sc}) - \mathcal{S}'(\varphi^{sc})(1 - \varphi^{sc}) = \mathcal{D}^{-1}(\varphi^{sc}) + \mathcal{S}(\varphi^{sc}).$$

Comparing (8) and (B2), one can deduce that  $\varphi^{sc} < \varphi^o$ , namely, *the planner elects a lower failure rate compared to the level that minimizes potential victims’ losses*. Thus, the attempted crime rate  $\alpha^{sc} > \alpha^o$  and individual vigilance  $\hat{\phi}^o(\varphi^{sc}|\xi) < \hat{\phi}^o(\varphi^o|\xi)$  for all  $\xi$ .

Could  $\varphi^{sc}$  be lower than its equilibrium counterpart  $\varphi^*$ ? Since the excess of supply  $\mathcal{S}(\varphi) - \mathcal{D}^{-1}(\varphi)$  vanishes when  $\varphi = \varphi^*$ , the social cost minimizer  $\varphi^{sc} < \varphi^*$  if and only if  $\varphi^{sc}$  induces excess of supply, or  $\mathcal{S}(\varphi^{sc}) - \mathcal{D}^{-1}(\varphi^{sc}) > 0$ . By (B2),

$$\mathcal{S}(\varphi^{sc}) - \mathcal{D}^{-1}(\varphi^{sc}) = \mathcal{S}(\varphi^{sc}) + \mathcal{S}'(\varphi^{sc})(1 - \varphi^{sc}) = \mathcal{S}(\varphi^{sc})(1 - \mathcal{E}_{1-\varphi}(\mathcal{S})).$$

Thus,  $\varphi^{sc}$  induces excess of supply if, at the optimum, the supply elasticity with respect to the success rate is low enough, i.e.,  $\mathcal{E}_{1-\varphi}(\mathcal{S}) < 1$ . Conversely,  $\varphi^{sc}$  induces excess of demand if this elasticity is high enough,  $\mathcal{E}_{1-\varphi}(\mathcal{S}) > 1$ .

**PROPOSITION B.1:** (i) If  $F$  is sufficiently concave, i.e.,  $F(c)/(c + \ell)$  is decreasing in  $c$ , then the social cost minimizer  $\varphi^{sc} < \varphi^*$ ; (ii) if  $F$  is convex, or not too concave, namely, if  $F(c)/(c + \ell)$  is increasing in  $c$ , the social cost minimizer  $\varphi^{sc} > \varphi^*$ .

**PROOF:**

For proving (i) and (ii), I exploit the fact that  $\mathcal{E}_{1-\varphi}(\mathcal{S}) = (\bar{c} + \ell)f(\bar{c})/F(\bar{c})$ , where  $\bar{c} = (1 - \varphi^{sc})M - \ell$ . Part (i): Log-differentiate  $F(c)/(c + \ell)$  in  $c$  to get  $\frac{f(c)}{F(c)} - \frac{1}{c + \ell} \leq 0$ , where the inequality follows since  $F(c)/(c + \ell)$  falls in  $c$ . Next, rearrange this inequality to obtain  $(c + \ell)f(c)/F(c) \leq 1$ . Thus,  $\mathcal{E}_{1-\varphi}(\mathcal{S}) \leq 1$ , and so  $\varphi^{sc} < \varphi^*$ .

Part (ii): If  $F$  is convex, the secant  $F(c)/c$  is increasing, and so  $(c + \ell)f(c)/F(c) \geq cf(c)/F(c) \geq 1$ , implying the desired conclusion. ■

APPENDIX C. THE OBSERVABLE VIGILANCE CASE

**CLAIM C.1:** The failure rate function  $\tilde{\phi}$  is submodular in  $(\phi, \xi)$ ; i.e.,  $\tilde{\phi}_{v\xi} \leq 0$ .

**PROOF:**

Log-differentiate  $\tilde{\phi}_v(v, \xi)$ , using (10), to get

$$\frac{\tilde{\phi}_{v\xi}}{\tilde{\phi}_v} = -\left(\frac{V_{\phi\phi}}{V_\phi} \tilde{\phi}_\xi + \frac{V_{\phi\xi}}{V_\phi}\right) = -\left(\frac{V_{\phi\phi} - V_\xi}{V_\phi} + \frac{V_{\phi\xi}}{V_\phi}\right) = -\frac{V_\xi}{V_\phi} \left(-\frac{V_{\phi\phi}}{V_\phi} + \frac{V_{\phi\xi}}{V_\xi}\right).$$

Now, notice that the parenthesized term above is positive,

$$-\frac{V_{\phi\phi}}{V_\phi} + \frac{V_{\phi\xi}}{V_\xi} = \left(\frac{V_\phi}{V} - \frac{V_{\phi\phi}}{V_\phi}\right) + \left(\frac{V_{\phi\xi}}{V_\xi} - \frac{V_\phi}{V}\right) \geq 0,$$

as  $V$  is log-concave in  $\phi$  and log-supermodular in  $(\phi, \xi)$ . Thus,  $\tilde{\phi}_{v\xi} \leq 0$ , as  $\tilde{\phi}_v, V_\xi, V_\phi > 0$ . ■

Before proving Proposition 7, slightly abuse notation and rewrite victims' losses (1) as

$$\mathcal{L}(\xi, \hat{\xi}, v | \tilde{\alpha}) \equiv \tilde{\alpha}(v, \hat{\xi})(1 - \tilde{\phi}(v, \xi))M + v$$

for  $v \geq 0$  and  $\xi, \hat{\xi} \in [0, 1]$ , and  $\tilde{\alpha}(\cdot) \geq 0$  with  $\tilde{\alpha}_v, \tilde{\alpha}_\xi \neq 0$ .

C1. Equilibrium Characterization: Proof of Proposition 7

**Step 1:** THE FAILURE RATE  $\tilde{\phi}(\chi(\xi), \xi)$  IS CONSTANT FOR ALL  $\xi \in [0, 1]$ . First, I show that there is no interval for which  $\tilde{\phi}$  is strictly monotone. By contradiction, suppose that, given  $\chi$ , the failure rate function  $\tilde{\phi}$  is strictly monotone for types  $\xi \in [\xi', \xi'']$  with  $\xi'' > \xi'$ . Without loss of generality, assume  $\tilde{\phi}$  is strictly increasing in this set. Then, if  $\xi'$  attracts no criminals, then so does  $\xi \in [\xi', \xi'']$ ; thus, all types

would want to mimic the type that chose the lowest vigilance level. Now, if  $\xi'$  does attract criminals, then any type  $\xi \in (\xi', \xi'']$  does not (as  $\tilde{\phi}$  is strictly increasing); therefore, any type  $\xi \in (\xi', \xi'']$  is better off imitating a type that attracts no criminal and elects vigilance strictly less than  $\chi(\xi)$ .

Now, I show that  $\tilde{\phi}(\chi(\xi), \xi)$  must be continuous in  $[0, 1]$ . Suppose that  $\tilde{\phi}$  is discontinuous, and call  $\xi_1 \in [0, 1]$  its first discontinuity point. As previously argued,  $\tilde{\phi}$  must be constant for all  $\xi \neq \xi_1$  in a neighborhood of  $\xi_1$ . Without loss of generality, suppose that  $\tilde{\phi}(\chi(\xi_1), \xi_1) > \tilde{\phi}(\chi(\xi_0), \xi_0)$  for  $\xi_0 \neq \xi_1$  in this neighborhood, and  $\xi_0$  arbitrarily close to  $\xi_1$ . Now, since  $\tilde{\phi}$  is continuous,  $\chi$  must be discontinuous at  $\xi_1$ , with  $\epsilon \equiv \chi(\xi_1) - \chi(\xi_0) > 0$ . Clearly, the losses of  $\xi_1$  are  $\chi(\xi_0) + \epsilon$ , whereas the losses for  $\xi_0$  are  $\tilde{\alpha}(\chi(\xi_0), \xi_0)(1 - \tilde{\phi}(\chi(\xi_0), \xi_0))M + \chi(\xi_0)$ . So, if  $\epsilon \geq \tilde{\alpha}(\chi(\xi_0), \xi_0)(1 - \tilde{\phi}(\chi(\xi_0), \xi_0))M$ ,  $\xi_1$  is better off imitating  $\xi_0$ ; otherwise,  $\xi_0$  is better off imitating  $\xi_1$ .

Altogether, there exists a failure rate  $\tilde{\varphi} \in (0, 1)$  with  $\chi(\xi) \equiv V(\tilde{\varphi}|\xi) > 0$  with  $\tilde{\phi}(\chi(\xi), \xi) = \tilde{\varphi}$  for all  $\xi \in [0, 1]$ .<sup>46</sup> As a by-product, vigilance  $\chi(\cdot)$  is strictly increasing in  $\xi$ , as  $V_\xi(\tilde{\varphi}|\cdot) > 0$ .

**Step 2: THE ATTEMPTED CRIME RATE FUNCTION  $\tilde{\alpha}$  SOLVES (12) AND THE ODE (11).** First, the ODE. In a separating equilibrium, incentive compatibility (13) requires  $\mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha}) \leq \mathcal{L}(\xi, \chi^{-1}(v), v | \tilde{\alpha})$  for all types  $\xi \in [0, 1]$ , and vigilance  $v \in \chi([0, 1])$ . But since  $\chi$  is one-to-one, for each  $v \in \chi([0, 1])$  there is a unique  $\hat{\xi} = \chi^{-1}(v) \in [0, 1]$ , and so (13) can be rewritten as

$$\xi \in \arg \min_{\hat{\xi} \in [0, 1]} \tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi}) \left( 1 - \tilde{\phi}(\chi(\hat{\xi}), \hat{\xi}) \right) M + \chi(\hat{\xi}),$$

for all  $\xi \in [0, 1]$ . Thus, the first-order condition, evaluated at  $\hat{\xi} = \xi$ , must hold:

$$\left[ \tilde{\alpha}_v(\chi(\xi), \xi) + \tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi) \frac{1}{\chi'(\xi)} \right] \left( 1 - \phi(\chi(\xi), \xi) \right) M - \tilde{\alpha}(\chi(\xi), \xi) \phi_v(\chi(\xi), \xi) M + 1 = 0.$$

Now, use (10) to substitute  $\phi_v$ , and multiply both sides of the first-order condition above by  $V_\phi(\tilde{\varphi}|\xi)$  to get (11), where  $\chi(\xi) = V(\tilde{\varphi}|\xi)$ . Finally, in equilibrium,  $\tilde{\alpha}$  and  $\tilde{\varphi}$  must be consistent with criminal optimization; namely, the market clearing condition (12) must hold.

<sup>46</sup> If  $\tilde{\varphi} = 0$ , then  $\chi(\xi) = 0$  for all  $\xi$ , and so a separating equilibrium cannot be sustained. Likewise, if  $\tilde{\varphi} = 1$ , all victims face no property losses; thus, each victim has incentives to deviate from  $\chi(\cdot) = V(1|\cdot)$ .



**Step 3:** THE ATTEMPTED CRIME RATE FUNCTION  $\tilde{\alpha}$  IS STRICTLY DECREASING. First, since  $\hat{\xi} = \xi$  minimizes expected losses  $\mathcal{L}(\xi, \hat{\xi}, \chi(\hat{\xi}) | \tilde{\alpha})$  over  $\hat{\xi} \in [0, 1]$ , the following second-order condition, evaluated at  $\hat{\xi} = \xi$ , must hold:

$$\begin{aligned} (\clubsuit) &= \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + 2\mathcal{L}_{v\hat{\xi}}(\xi, \xi, \chi(\xi))\chi'(\xi) + \mathcal{L}_{\hat{\xi}}(\xi, \xi, \chi(\xi))\chi''(\xi) \\ &\quad + \mathcal{L}_{vv}(\xi, \xi, \chi(\xi)) [\chi'(\xi)]^2 \geq 0. \end{aligned}$$

Also, the FOC holds for all  $\xi \in [0, 1]$ , and so  $\mathcal{L}_{\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_v(\xi, \xi, \chi(\xi))\chi'(\xi) \equiv 0$ . Differentiating the identity above with respect to  $\xi$  yields:

$$(\clubsuit) + \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_{v\xi}(\xi, \xi, \chi(\xi))\chi'(\xi) = 0.$$

Therefore, since  $(\clubsuit) \geq 0$  by the second-order conditions,

$$(C1) \quad 0 \geq \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_{v\xi}(\xi, \xi, \chi(\xi))\chi'(\xi),$$

where

$$\begin{aligned} \mathcal{L}_{\hat{\xi}\hat{\xi}}(\xi, \xi, \chi(\xi)) + \mathcal{L}_{v\xi}(\xi, \xi, \chi(\xi))\chi'(\xi) &= -[\tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi) \tilde{\phi}_{\xi}(\chi(\xi), \xi) \\ &\quad + \tilde{\alpha}_v(\chi(\xi), \xi) \phi_{\xi}(\chi(\xi), \xi)\chi'(\xi) \\ &\quad + \tilde{\alpha}(\chi(\xi), \xi) \phi_{v\xi}(\chi(\xi), \xi)\chi'(\xi)]. \end{aligned}$$

But since  $\tilde{\phi}_{v\xi} < 0 < \chi'$ , inequality (C1) implies

$$\tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi) \tilde{\phi}_{\xi}(\chi(\xi), \xi) + \tilde{\alpha}_v(\chi(\xi), \xi) \phi_{\xi}(\chi(\xi), \xi)\chi'(\xi) > 0.$$

Finally, because  $\tilde{\phi}_{\xi} < 0$ , the inequality immediately above reduces to

$$\frac{d\tilde{\alpha}(\chi(\xi), \xi)}{d\xi} = \tilde{\alpha}_v(\chi(\xi), \xi)\chi'(\xi) + \tilde{\alpha}_{\hat{\xi}}(\chi(\xi), \xi) < 0.$$

That is,  $\tilde{\alpha}(\chi(\xi), \xi)$  must be strictly decreasing in  $\xi$ .

**Step 4:** THE ATTEMPTED CRIME RATE FUNCTION  $\tilde{\alpha}$  VANISHES AT  $\xi = 1$ . By contradiction, suppose  $\tilde{\alpha}(\chi(1), 1) > 0$ . If type  $\xi = 1$  deviates and chooses  $v^d = \chi(1) + \epsilon$ , with  $0 < \epsilon < \tilde{\alpha}(\chi(1), 1)(1 - \tilde{\varphi})M$ , then potential criminals, after observing  $v^d$ , would expect an associated failure rate that is higher than the equilibrium one, since for any inference  $\hat{\xi}$ ,

$$\tilde{\phi}(v^d, \hat{\xi}) \geq \tilde{\phi}(v^d, 1) = \tilde{\phi}(\chi(1) + \epsilon, 1) > \tilde{\phi}(\chi(1), 1) = \tilde{\varphi}.$$

Thus, no criminal would target potential victims choosing  $v^d$ . This deviation is also profitable, as  $\mathcal{L}(1, 1, \chi(1) | \tilde{\alpha}) > \chi(1) + \epsilon$ . Thus, in equilibrium,  $\tilde{\alpha}(\chi(1), 1) = 0$ . ■

C2. The Envelope Formula and Sufficient Conditions

CLAIM C.2: Consider an equilibrium  $(\chi(\cdot), \tilde{\alpha}(\cdot))$ . Then, potential victims' equilibrium losses  $\mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha})$  are strictly increasing in type  $\xi$  and obey the envelope formula:

$$(C2) \quad \mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha}) = \chi(1) + \int_{\xi}^1 \tilde{\alpha}(\chi(t), t) \tilde{\phi}_{\xi}(\chi(t), t) M dt.$$

PROOF:

First, since  $\tilde{\phi}(v, \cdot)$  is differentiable for all  $v > 0$ , it follows that  $\mathcal{L}_{\xi}(\xi, \hat{\xi}, \chi(\hat{\xi})) = -\tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi}) \tilde{\phi}_{\xi}(\chi(\hat{\xi}), \hat{\xi}) M < -\tilde{\alpha}(\chi(0), 0) \tilde{\phi}_{\xi}(\chi(1), \xi) M$ , where the inequality holds as  $\tilde{\alpha}$  is strictly decreasing (Proposition 7),  $\tilde{\phi}_{\xi}(v, \xi)$  strictly decreasing in  $v$  (Claim C.1), and  $\chi' > 0$  (Proposition 7). Next,  $\int_0^1 -\tilde{\alpha}(\chi(0), 0) \tilde{\phi}_{\xi}(\chi(1), \xi) d\xi = \tilde{\alpha}(\chi(0), 0) [\tilde{\phi}(\chi(1), 0) - \tilde{\phi}(\chi(1), 1)] < \infty$ . Finally, the envelope formula (C2) holds, by theorem 2 in Milgrom and Segal (2002). ■

PROPOSITION C.1: Consider a pair  $(\chi, \tilde{\alpha})$ , satisfying the necessary conditions of Proposition 7. If, in addition,  $(\chi, \tilde{\alpha})$  obeys formula (C2), the candidate pair  $(\chi, \tilde{\alpha})$  is an equilibrium.

PROOF:

It is enough to show that, given  $(\chi, \tilde{\alpha})$ , potential victims have no incentives to deviate on the equilibrium path. Pick arbitrary  $\xi, \hat{\xi} \in [0, 1]$ . Using formula (C2),

$$\begin{aligned} \mathcal{L}(\xi, \hat{\xi}, \chi(\hat{\xi}) | \tilde{\alpha}) &= \mathcal{L}(\hat{\xi}, \hat{\xi}, \chi(\hat{\xi}) | \tilde{\alpha}) + \tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi}) [\tilde{\phi}(\chi(\hat{\xi}), \hat{\xi}) - \tilde{\phi}(\chi(\hat{\xi}), \xi)] M \\ &= \mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha}) - \int_{\xi}^{\hat{\xi}} \tilde{\alpha}(\chi(t), t) \tilde{\phi}_{\xi}(\chi(t), t) M dt \\ &\quad + \tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi}) [\tilde{\phi}(\chi(\hat{\xi}), \hat{\xi}) - \tilde{\phi}(\chi(\hat{\xi}), \xi)] M \\ &= \mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha}) \\ &\quad - \underbrace{\int_{\xi}^{\hat{\xi}} \tilde{\alpha}(\chi(t), t) \tilde{\phi}_{\xi}(\chi(t), t) M dt + \tilde{\alpha}(\chi(\hat{\xi}), \hat{\xi}) \int_{\xi}^{\hat{\xi}} \tilde{\phi}_{\xi}(\chi(\hat{\xi}), t) M dt}_{(\star)}. \end{aligned}$$

Notice that  $(\star) \geq 0$  because the map  $t \mapsto \tilde{\alpha}(\chi(t), t) \tilde{\phi}_{\xi}(\chi(t), t)$  is increasing for all  $t' \in [0, 1]$ , by inequality (C1). Thus,  $\mathcal{L}(\xi, \xi, \chi(\xi) | \tilde{\alpha}) \leq \mathcal{L}(\xi, \hat{\xi}, \chi(\hat{\xi}) | \tilde{\alpha})$ , and so  $\chi$  is incentive compatible given  $\tilde{\alpha}$ ; i.e., condition (13) holds. ■

## APPENDIX D. VIGILANCE AND POLICING AS COMPLEMENTS

The baseline model assumes that the events in which the criminal fails and the criminal is captured are statistically independent. I now relax this assumption. Suppose that the probability of capture is  $p_h$  if the criminal fails (e.g., an alarm turned on and police became more aware of the incident) and  $p_l < p_h$  if the criminal succeeds. (The baseline model assumes  $p_h = p_l = p$ .) Recalling that  $x$  denotes the legal penalty when the criminal is apprehended, the expected criminal payoff (2) turns to

$$\varphi[-p_h x + (1 - p_h)0] + (1 - \varphi)[p_l(M - x) + (1 - p_l)M] - c,$$

which is equal to

$$(1 - \varphi)M - (\varphi p_h + (1 - \varphi)p_l)x - c,$$

where  $\varphi p_h + (1 - \varphi)p_l$  reflects the *expected probability of capture*. All told, the supply curve (5) turns to

$$\mathcal{S}(\varphi) = F[(1 - \varphi)M - (\varphi p_h + (1 - \varphi)p_l)x].$$

Differentiating  $\mathcal{S}$  in  $\varphi$  shows that the supply curve remains downward sloping:

$$\mathcal{S}'(\varphi) = F'[\cdot](-M - (p_h - p_l)x) = -F'[\cdot](M + (p_h - p_l)x) < 0.$$

Thus, the qualitative findings of this paper remain valid. Now, one may wonder about the effects on the *levels* of crime and vigilance. If apprehending criminals is more likely when the vigilance and policing are complements, i.e.,  $\varphi p_h + (1 - \varphi)p_l > p$ , then the new supply curve is below the one in which there are no complementarities because

$$F[(1 - \varphi)M - (\varphi p_h + (1 - \varphi)p_l)x] < F[(1 - \varphi)M - px].$$

Because demand  $\mathcal{D}$  is upward sloping, the new equilibrium entails fewer attempted crimes  $\alpha$  but a greater success rate  $1 - \varphi$ , relative to the benchmark case. The crime rate  $\kappa = (1 - \varphi)\alpha$  could rise or fall depending on the demand elasticity, as discussed in Section IV. Altogether, the resulting model would be isomorphic to a model with no complementarities but with a higher probability of capture (i.e., higher  $\ell$ ).

## REFERENCES

- Anderson, David A. 2011. "The Cost of Crime." *Foundations and Trends in Microeconomics* 7 (3): 209–65.
- Andreoni, James, Brian Erard, and Jonathan Feinstein. 1998. "Tax Compliance." *Journal of Economic Literature* 36 (2): 818–60.
- Armstrong, Mark, John Vickers, and Jidong Zhou. 2009. "Consumer Protection and the Incentive to Become Informed." *Journal of the European Economic Association* 7 (2–3): 399–410.
- Athey, Susan. 2002. "Monotone Comparative Statics under Uncertainty." *Quarterly Journal of Economics* 117 (1): 187–223.

- Ayres, Ian. 2016. "Contracting for Privacy Precaution (and a Laffer Curve for Crime)." *Journal of Legal Studies* 45 (S2): S123–36.
- Ayres, Ian, and Steven D. Levitt. 1998. "Measuring Positive Externalities from Unobservable Victim Precaution: An Empirical Analysis of Lojack." *Quarterly Journal of Economics* 113 (1): 43–77.
- Bagnoli, Mark, and Ted Bergstrom. 2005. "Log-Concave Probability and Its Applications." *Economic Theory* 26 (2): 445–69.
- Baumann, Florian, Philipp Denter, and Tim Friehe. 2019. "Hide or Show? Observability of Private Precautions against Crime When Property Value Is Private Information." *American Law and Economics Review* 21 (1): 209–45.
- Baumann, Florian, and Tim Friehe. 2013. "Private Protection against Crime When Property Value Is Private Information." *International Review of Law and Economics* 35: 73–79.
- Becker, Gary S. 1968. "Crime and Punishment: An Economic Approach." *Journal of Political Economy* 76 (2): 169–217.
- Becker, Gary S., and George J. Stigler. 1974. "Law Enforcement, Malfeasance, and Compensation of Enforcers." *Journal of Legal Studies* 3 (1): 1–18.
- Ben-Shahar, Omri, and Alon Harel. 1995. "Blaming the Victim: Optimal Incentives for Private Precautions Against Crime." *Journal of Law, Economics, and Organization* 11 (2): 434–55.
- Borker, Girija. 2018. "Safety First: Perceived Risk of Street Harassment and Educational Choices of Women." [https://data2x.org/wp-content/uploads/2019/11/PerceivedRiskStreetHarassmentandEdu-ChoicesofWomen\\_Borker.pdf](https://data2x.org/wp-content/uploads/2019/11/PerceivedRiskStreetHarassmentandEdu-ChoicesofWomen_Borker.pdf).
- Burdett, Kenneth, Ricardo Lagos, and Randall Wright. 2003. "Crime, Inequality, and Unemployment." *American Economic Review* 93 (5): 1764–77.
- Chalfin, Aaron, and Justin McCrary. 2017. "Criminal Deterrence: A Review of the Literature." *Journal of Economic Literature* 55 (1): 5–48.
- Choi, Michael, and Lones Smith. 2017. "Ordinal Aggregation Results via Karlin's Variation Diminishing Property." *Journal of Economic Theory* 168: 1–11.
- Clotfelter, Charles T. 1977. "Public Services, Private Substitutes, and the Demand for Protection Against Crime." *American Economic Review* 67 (5): 867–77.
- Clotfelter, Charles T. 1978. "Private Security and the Public Safety." *Journal of Urban Economics* 5 (3): 388–402.
- Cook, Philip J. 1986. "The Demand and Supply of Criminal Opportunities." *Crime and Justice* 7: 1–27.
- Cook, Philip J., and John MacDonald. 2011. "Public Safety Through Private Action: An Economic Assessment of BIDS." *Economic Journal* 121 (552): 445–62.
- Dominguez, Patricio. 2020. "How Potential Offenders and Victims Interact." Inter-American Development Bank Working Paper DB-WP-1108.
- Draca, Mirko, Theodore Koutzeridis, and Stephen Machin. 2019. "The Changing Returns to Crime: Do Criminals Respond to Prices?" *Review of Economic Studies* 86 (3): 1228–57.
- Draca, Mirko, and Stephen Machin. 2015. "Crime and Economic Incentives." *Annual Review of Economics* 7: 389–408.
- Ehrlich, Isaac. 1981. "On the Usefulness of Controlling Individuals: An Economic Analysis of Rehabilitation, Incapacitation, and Deterrence." *American Economic Review* 71 (3): 307–22.
- Ehrlich, Isaac. 2010. "The Market Model of Crime: A Short Review and New Directions." In *Handbook on the Economics of Crime*, edited by Bruce L. Benson and Paul R. Zimmerman, 3–23. Northampton, MA: Edward Elgar Publishing.
- Farrell, Graham. 2016. "Attempted Crime and the Crime Drop." *International Criminal Justice Review* 26 (1): 21–30.
- Farrell, Graham, Nick Tilley, and Andromachi Tseloni. 2014. "Why the Crime Drop?" *Crime and Justice* 43 421–90.
- Farrell, Graham, Andromachi Tseloni, Jen Mailley, and Nick Tilley. 2011. "The Crime Drop and the Security Hypothesis." *Journal of Research in Crime and Delinquency* 48 (2): 147–75.
- Fujita, Shuryo, and Michael Maxfield. 2012. "Security and the Drop in Car Theft in the United States." In *The International Crime Drop: New Directions in Research*, edited by Jan J. Van Dijk, Andromachi Tseloni, and Graham Farrell, 231–49. London: Palgrave Macmillan.
- Garoupa, Nuno. 2003. "Optimal Law Enforcement When Victims are Rational Players." In *Conflict and Governance*, edited by Amihai Glazer and Kai A. Konrad, 123–34. Berlin: Springer.
- Gonzalez-Navarro, Marco. 2013. "Deterrence and Geographical Externalities in Auto Theft." *American Economic Journal: Applied Economics* 5 (4): 92–110.
- Graetz, Michael J., Jennifer F. Reinganum, and Louis L. Wilde. 1986. "The Tax Compliance Game: Toward an Interactive Theory of Law Enforcement?" *Journal of Law, Economics, and Organization* 2 (1): 1–32.

- Helsley, Robert W., and William C. Strange.** 2005. "Mixed Markets and Crime." *Journal of Public Economics* 89 (7): 1251–75.
- Hotte, Louis, Fabrice Valognes, and Tanguy van Ypersele.** 2003. "Property Crime with Private Protection: A Market-for-Offenses Approach." CEPR Discussion Paper 3782.
- Hotte, Louis, and Tanguy Van Ypersele.** 2008. "Individual Protection against Property Crime: Decomposing the Effects of Protection Observability." *Canadian Journal of Economics* 41 (2): 537–63.
- Hylton, Keith N.** 1996. "Optimal Law Enforcement and Victim Precaution." *RAND Journal of Economics* 27 (1): 197–206.
- Karlin, Samuel.** 1968. *Total Positivity, Vol. I*. Stanford, CA: Stanford University Press.
- Knowles, John, Nicola Persico, and Petra Todd.** 2001. "Racial Bias in Motor Vehicle Searches: Theory and Evidence." *Journal of Political Economy* 109 (1): 203–29.
- Koo, Hui-wen, and I.P.L. Png.** 1994. "Private Security: Deterrent or Diversion?" *International Review of Law and Economics* 14 (1): 87–101.
- Lacroix, Guy, and Nicolas Narceau.** 1995. "Private Protection Against Crime." *Journal of Urban Economics* 37 (1): 72–87.
- Landes, William M., and Richard A. Posner.** 1975. "The Private Enforcement of Law." *Journal of Legal Studies* 4 (1): 1–46.
- Makridis, Christos, and Benjamin Dean.** 2018. "Measuring the Economic Effects of Data Breaches on Firm Outcomes: Challenges and Opportunities." *Journal of Economic and Social Measurement* 43 (1–2): 59–83.
- McAfee.** 2018. *Economic Impact of Cybercrime—No Slowing Down*. Santa Clara, CA: McAfee.
- Milgrom, Paul, and Ilya Segal.** 2002. "Envelope Theorems for Arbitrary Choice Sets." *Econometrica* 70 (2): 583–601.
- Nagin, Daniel S.** 2013. "Deterrence: A Review of the Evidence by a Criminologist for Economists." *Annual Review of Economics* 5: 83–105.
- O’Flaherty, Brendan, and Rajiv Sethi.** 2008. "Racial Stereotypes and Robbery." *Journal of Economic Behavior and Organization* 68 (3–4): 511–24.
- Persico, Nicola.** 2002. "Racial Profiling, Fairness, and Effectiveness of Policing." *American Economic Review* 92 (5): 1472–97.
- Polinsky, A. Mitchell, and Steven Shavell.** 2000. "The Economic Theory of Public Enforcement of Law." *Journal of Economic Literature* 38 (1): 45–76.
- Prékopa, András.** 1973. "On Logarithmic Concave Measures and Functions." *Acta Scientiarum Mathematicarum* 34: 334–43.
- Priks, Mikael.** 2015. "The Effects of Surveillance Cameras on Crime: Evidence from the Stockholm Subway." *Economic Journal* 125 (588): F289–305.
- Quercioli, Elena, and Lones Smith.** 2015. "The Economics of Counterfeiting." *Econometrica* 83 (3): 1211–36.
- Shavell, Steven.** 1991. "Individual Precautions to Prevent Theft: Private versus Socially Optimal Behavior." *International Review of Law and Economics* 11 (2): 123–32.
- Topkis, Donald M.** 1998. *Supermodularity and Complementarity*. Princeton, NJ: Princeton University Press.
- Tseloni, Andromachi, Rebecca Thompson, Louise Grove, Nick Tilley, and Graham Farrell.** 2017. "The Effectiveness of Burglary Security Devices." *Security Journal* 30 (2): 646–64.
- Tullock, Gordon.** 1967. "The Welfare Costs of Tariffs, Monopolies, and Theft." *Economic Inquiry* 5 (3): 224–32.
- Tullock, Gordon.** 1980. "Efficient Rent Seeking." In *Toward a Theory of the Rent-Seeking Society*, edited by James M. Buchanan, Robert D. Tollison and Gordon Tullock, 97–112. College Station, TX: Texas A&M University Press.
- van Ours, Jan C., and Ben Vollaard.** 2015. "The Engine Immobiliser: A Non-starter for Car Thieves." *Economic Journal* 126 (593): 1264–91.
- Vollaard, Ben, and Pierre Koning.** 2009. "The Effect of Police on Crime, Disorder, and Victim Precaution: Evidence from a Dutch Victimization Survey." *International Review of Law and Economics* 29 (4): 336–48.
- Vollaard, Ben, and Jan C. van Ours.** 2011. "Does Regulation of Built-in Security Reduce Crime? Evidence from a Natural Experiment." *Economic Journal* 121 (552): 485–504.
- Zimmerman, Paul R.** 2014. "The Deterrence of Crime through Private Security Efforts: Theory and Evidence." *International Review of Law and Economics* 37: 66–75.