On the Intensity of Enforcement and Crime*

Jorge Vásquez

June 9, 2025

Abstract

This paper develops a model of decentralized law enforcement to study the joint determination of enforcement intensity, criminal participation, and offense severity. A key feature of the model is that the probability of detection depends not only on enforcement effort but also on crime severity, reflecting institutional considerations such as citizen reporting and discretionary policing. This approach generates rich feedback effects that jointly determine equilibrium crime rates, arrest rates, enforcement levels, and offense severity. I use the model to evaluate the impact of several policy interventions—including harsher penalties, improved outside options, enhanced detection technologies, and changes in enforcement compensation. While enforcement discretion can sometimes blunt the effectiveness of deterrence policies, I identify a key condition—log-modularity of the detection probability—that ensures these interventions lead to socially desirable outcomes. The model not only helps rationalize recent empirical findings but also yields new testable predictions about the relationship between enforcement effort, crime severity, and arrests.

JEL codes: K42, D40, C72.

Keywords: decentralized enforcement, offense severity, crime rate, arrest rate

*Smith College, Department of Economics, jvasquez@smith.edu. I thank Steven Durlauf, Gabriela Monahova, Lones Smith, Marek Weretka, and Mariyana Zapryanova for their comments and suggestions. All errors are my own.

1 Introduction

Law enforcement in many countries is highly decentralized; for example, in the US, policing responsibility is spread across local, county, and state agencies (Owens, 2020).[1] This decentralization is usually combined with police discretion over enforcement practices, which has been shown to significantly shape enforcement outcomes (Gonçalves and Mello, 2023; Weisburst, 2024). Moreover, recent evidence indicates that police behavior often aligns more closely to tangible objectives—such as maximizing arrests or meeting implicit quotas—than to purely prevent crime (Stashko, 2022; Ossei-Owusu, 2021).[2] Consequently, the combination of decentralized and discretionary enforcement, with arrest-driven incentives may substantially influence the effectiveness of policies aimed at reducing crime.

In this paper, I develop a model of decentralized law enforcement to explore how different policy interventions influence both enforcement efforts and criminal behavior. Traditionally, economic models of crime assume a centralized agency (e.g., a social planner) responsible for setting penalties and determining the probability of apprehension, while potential criminals face a binary decision: commit a crime or abstain (Becker, 1968). In contrast, I introduce a richer framework in which potential offenders decide not only whether to commit a crime but also the severity of their offense. Indeed, many criminal activities involve clear intensive margins: burglars choose the size of the loot, speeders decide how fast to drive, counterfeiters determine how much fake currency to produce, drug dealers select the quantity of drugs to transport, and even minor infractions such as illegal parking involve meaningful decisions regarding the extent of the offense. Central to these examples is the idea that *offense severity could influence the likelihood of detection*. A more severe offense tends to attract greater visibility and public attention, is often more difficult to conceal, and generally leaves behind more extensive evidence trails, thereby facilitating detection by law enforcement.[3]

The model rests on four main tenets: (i) law enforcement is decentralized; (ii) enforcement efforts aim at maximizing arrests; (iii) potential criminals strategically select the severity ("size") of their offenses; and (iv) the detection of criminals depend jointly on enforcement

[1]As of 2017, the US had around 22,800 enforcement agencies, including municipal police departments, sheriff's departments, county police departments, and state police agencies; see Owens (2020) for more details.

[2]Intuitively, arrests are easily quantifiable and thus readily integrated into performance evaluations, whereas crime prevention is inherently difficult to measure. Nevertheless, police agencies might still prioritize crime deterrence, depending on the context. For instance, Eeckhout et al. (2010) analyze administrative records from Belgian police departments, providing evidence that their primary objective in issuing traffic tickets is to deter speeding, not to maximize revenue (pp. 1116–1117).

[3]As noted in Quercioli and Smith (2015), in an interview with NPR's *All Things Considered*, Kersten remarks: "One of the things that made him (counterfeiter Art Williams) successful is that he limited his production. If a counterfeiter goes out there and prints a million dollars, he's going to get caught right away because when you flood the market with that much fake currency, the Secret Service is going to be all over you very quickly." (Kersten, 2005).

effort and crime severity. Together, these tenets generate novel feedback effects between law enforcers and potential offenders, influencing equilibrium outcomes such as enforcement levels, crime severity, crime rates, and arrest rates. I examine the impact of various policy interventions, including harsher penalties, improved outside options for criminals, enhanced detection technologies, and adjustments in police compensation schemes. Although enforcement efforts typically fall in response to reductions in crime—potentially leading to unintended consequences—I identify clear sufficient conditions, particularly on the detection probability, that mitigate these adverse effects and help reconcile recent empirical findings. Furthermore, the resulting feedback effects can be understood through the familiar lens of supply-and-demand, augmented by strategic decisions regarding the severity of crimes.

To capture the decentralized and discretionary nature of law enforcement, I consider a continuum of law enforcers, each electing their costly enforcement effort.[4] I assume law enforcers are atomless, and thus individually unable to affect the overall crime rate; instead, each enforcer takes the crime rate as given and chooses their enforcement effort to maximize arrests. Operationally, I implement this using a random matching model: law enforcers encounter opportunities to apprehend criminals as a function of the prevailing crime rate. Arrests then occur probabilistically, reflecting real-world imperfections in detection and apprehension.[5] On the other side of the market, there is a continuum of heterogeneous potential criminals each choosing whether to commit a crime, and, if so, the severity of their offense. A more severe crime yields a greater reward but faces a higher probability of apprehension all else equal. In equilibrium, all players simultaneously best respond to one another.

To characterize equilibrium outcomes, I decompose the analysis into two interconnected components: (i) an induced market governed by supply-and-demand forces, and (ii) the optimal choice of crime severity. Equilibrium requires that the enforcement level clears the induced market for crime, while criminals simultaneously select an offense severity that maximizes their profits given this market-clearing enforcement level. To illustrate, fix the severity of offenses: potential criminals' decisions on whether or not to commit crimes generate a downward-sloping supply curve, since lower enforcement induces more individuals to engage in crime. Conversely, law enforcers' decisions create an upward-sloping "demand" curve, as a higher crime rate elicits more enforcement effort. The intersection of these curves defines the *market clearing locus*, mapping each offense severity to its corresponding market-clearing

[4]Recent estimates suggest that the US spends approximately $150 billion annually on police protection, a figure that has risen by about 21% over the past decade (Anderson, 2021).

[5]Many crimes go unreported to law enforcement agencies. According to the Bureau of Justice Statistics, approximately 30% of property crime victimizations were reported to police in 2023; see https://bjs.ojp.gov/library/publications/criminal-victimization-2023. Additionally, not all reported crimes result in arrests. In 2023, 41.1% of reported violent crimes and 13.9% of reported property crimes were cleared by arrest or exceptional means; see https://cde.ucr.cjis.gov/LATEST/webapp/#/pages/home.

enforcement level. Under the assumption that the detection probability is supermodular in enforcement effort and offense severity, this locus slopes upward. Considering the criminals' optimal severity choice separately, this one decreases with higher enforcement. Thus, equilibrium is determined by the intersection of two loci: an upward-sloping market clearing locus and a downward-sloping severity locus. This intersection characterizes a unique equilibrium, yielding a rich yet tractable framework suitable for comparative statics (Proposition 1).

In Proposition 2, I analyze the effects of imposing harsher punishments. Stricter penalties deter more potential criminals, and those who remain undeterred commit less severe offenses. Thus, because enforcement is costly, this leads to an unambiguous decrease in equilibrium enforcement level. However, the overall impact on crime severity, the crime rate, and the arrest rate remain ambiguous. A key contribution is identifying conditions on the detection probability that ensure desirable policy outcomes. Proposition 2 establishes that such favorable results emerge when the detection probability $\wp(e, \sigma)$ is log-modular in enforcement effort e and crime severity σ. For instance, any Cobb-Douglas function, such as $\wp(e, \sigma) = e^a \sigma^b$ with $a, b \in \mathbb{R}$, satisfies this property. Under this condition, harsher punishments reduce enforcement levels but still lead to fewer arrests, lower offense severity, and a lower crime rate. Importantly, this log-modularity condition is not merely a technical artifact but reflects a practical institutional mechanism: citizens report offenses as a function of crime severity, and conditional on such reports, police exert effort to apprehend offenders.

The landmark paper by Becker (1968) predicts that better employment opportunities—or improved outside options—should reduce overall crime. However, it is less clear how changes in outside options affect the severity of offenses committed by those who remain criminally active. Proposition 3 shows that while improved outside options effectively discourages crime participation, they also lead to lower enforcement effort, which in turn increases offense severity. The intuition is that as crime falls, enforcers reduce their effort, raising the marginal returns to more severe offenses. As a result, fewer individuals commit crimes, but those who do engage in more severe offenses. This insight helps reconcile recent empirical findings showing that negative income shocks can affect not just the likelihood of offending, but also the intensity of criminal behavior (e.g., Bignon et al., 2015; Giulietti and McConnell, 2024).

Recent empirical studies indicate that the probability of detection is influenced by environmental or technological factors (e.g., Vollaard, 2017; Anker et al., 2021), generally supporting the view that an increased probability of detection reduces crime. However, theoretical implications are more nuanced, as detection probabilities simultaneously affect both sides of the market—law enforcers and potential criminals—leading to ambiguous effects on equilibrium outcomes. Proposition 4 provides conditions that resolve this ambiguity, helping to reconcile empirical findings while offering new testable implications. Specifically, if the detection probability is, again, log-modular in enforcement and offense severity, then a proportional increase in detection probability leads to desirable outcomes: fewer crimes, reduced severity of offenses, and fewer arrests—all achieved without requiring higher enforcement.

The final policy exercise examines the effects of strengthening the incentives for law enforcers to apprehend criminals. Specifically, what occurs when law enforcers are more strongly rewarded for arrests? Such a change indirectly influences potential criminals through adjustments in enforcement effort. Naturally, a greater reward for arresting criminals encourages increased enforcement, deterring criminal entry and reducing the severity of committed offenses. However, the overall effect on arrests is more subtle because the rise in enforcement is accompanied by a decrease in criminal activity. Proposition 5 shows that, under mild conditions, the equilibrium arrest rate actually declines. This finding implies that using arrest data as a proxy for enforcement intensity should be approached with caution.

Finally, I explore several extensions and robustness checks. First, I consider a centralized enforcement setting in which enforcement effort directly impacts the crime rate. I find that centralization—coupled with arrest-driven incentives—produce undesirable outcomes: crime rates and offense severity both increase, while equilibrium enforcement declines (Proposition 6). Second, I extend the results to settings where the penalty for offenses explicitly depends on their severity. Lastly, I show that while the baseline model assumes supermodularity of the detection probability $\wp(e, \sigma)$, relaxing this to allow for submodularity can be easily accommodated; moreover, such extension could help explain how punishment and enforcement could act as complements, as suggested in recent empirical work (Soliman, 2022).

The rest of the paper proceeds as follows. Section §2 reviews the related literature. Section §3 introduces the baseline model, and Section §4 characterizes equilibrium outcomes. Section §5 provides a fully solved example, and Section §6 explores policy interventions. Section §7 examines extensions and variations to the baseline model. Finally, Section §8 concludes. Omitted proofs are provided in the Appendix.

2 Related literature

This paper relates to the literature on illegal behavior and public enforcement of laws, pioneered by Becker (1968) and surveyed by Garoupa (1997) and Polinsky and Shavell (2000).[6] This research line typically studies the decision problem of a central authority that jointly chooses the probability of detection and the legal penalty to optimize social welfare, accounting for criminals' optimal responses. My paper differs in several important ways. First, it

[6] For a clear and concise overview of Becker's approach to crime and deterrence, along with a comprehensive survey of the empirical literature it inspired, see Chalfin and McCrary (2017).

separates the choice of enforcement from the choice of punishment, reflecting that in practice law enforcers typically take penalties as given and independently select their enforcement intensity. Second, following Di Porto et al. (2013), I adopt a decentralized approach with many enforcers making individual decisions rather than a single centralized authority.[7,8]

By allowing offenders to make marginal decisions regarding crime severity, my paper also connects to the literature on marginal deterrence (Stigler, 1970; Shavell, 1992; Mookherjee and Png, 1994) which primarily focuses on sanction design. My analysis examines a critical yet often overlooked trade-off: crime severity directly influences the detection probability.[9] This generates rich feedback mechanisms, determining crime rates, arrest rates, enforcement intensity, and crime severity as equilibrium outcomes.

Relatedly, a smaller literature explores interactions between enforcers and criminals where offenders invest in costly detection avoidance (e.g., Malik, 1990; Friehe and Miceli, 2017). In this literature, the detection probability depends on both enforcement and avoidance efforts, providing an argument against maximal penalties since higher fines naturally induce greater avoidance efforts by criminals. In contrast, my paper examines a broader set of policy interventions aimed at reducing crime and identifies sufficient conditions on the detection probability function that yield desirable policy outcomes.

Finally, there is a growing literature leveraging random matching models where individuals take costly actions to prevent harm (e.g., Quercioli and Smith, 2015; Vásquez, 2022; Cisternas and Vásquez, 2022). These models generate induced supply-and-demand frameworks useful for analyzing policy interventions. However, the combination of heterogeneous potential criminals making both extensive (whether to commit a crime) and intensive (severity of crime) decisions, alongside multiple enforcers independently choosing enforcement intensity, implies that equilibrium cannot be fully characterized with a supply-and-demand approach. Instead, the supply-and-demand framework only partially determines equilibrium outcomes, with optimal crime severity decisions providing the complementary piece needed for a complete equilibrium characterization.

[7] Di Porto et al. (2013) develops a tax evasion where firms report their private income to auditors, taking into account audit probabilities and penalties. They note, "[. . .] enforcement is not actually carried out by a unitary actor, but by a multitude of individual 'auditors' (police officers, tax inspectors, etc.) whose individual behavior has negligible impact but whose aggregate behavior generates deterrence" (p. 35).

[8] Assuming centralized enforcement, a related literature focuses on the design of policing strategies. Eeckhout et al. (2010) show how random, publicly announced crackdowns can arise optimally from police incentives. More recently, Gao and Vásquez (2024) studies optimal allocations of police resources across heterogeneous crime targets when criminal can engage in sequential search.

[9] The conventional view suggests that if penalties are independent of severity, criminals lack incentives to reduce crime intensity (Stigler, 1970). However, if crime severity affects the detection probability, then criminals may have incentives to scale back their intensity even without severity-dependent penalties.

3 The Model

Players. There is a unit mass of *law enforcers*, and a large mass of *potential criminals*. Potential criminals differ in their *outside option* $\omega \in [0, \bar{w}]$, with $\bar{w} \in \overline{\mathbb{R}}_+$ (extended positive reals), which represents, e.g., legal earning opportunities. Outside options are distributed according to an atomless cumulative distribution function $G(\omega)$ with density $g(\omega) \equiv dG(\omega)/d\omega > 0$ on $(0, \bar{w})$. When ω's support is bounded, i.e., $\bar{w} < \infty$, it is assumed that $\bar{w}$ is high enough (to be made precise shortly) so that there is always a non-negligible mass of potential criminals unwilling to engage in crime.

Actions and Capture. Potential criminal chooses whether to commit a crime and, if so, they also elect the *severity* $\sigma \in [0,1]$ of their offense. Henceforth, potential criminals who choose to engage in crime are referred to simply as *criminals*. On the other hand, law enforcers choose a costly *enforcement level* $e \in [0,1]$ to capture criminals (e.g., policing or patrolling). However, capturing a criminal requires first encountering one. This is modeled with a random matching technology: law enforcers randomly encounter criminals at rate $\kappa \geq 0$, referred to as the the *crime rate*, which is determined endogenously by the mass of potential criminals who choose to engage in crime. The *arrest rate* $\alpha \in [0,1]$ denotes the mass of criminals who are successfully apprehended. Intuitively, each law enforcer is matched with a "criminal case" at rate κ and successfully "clears" the case at rate α. The detection technology—i.e., the probability of apprehension conditional on a match—is described next.

As in Becker (1968), a potential criminal who commits a crime may not be caught. I assume the enforcement level e and crime severity σ fix the *capture chance* $\wp(e, \sigma)$, where $\wp : [0,1]^2 \to [0,1]$ is a twice continuously differentiable function. Thus, the arrest rate is simply $\alpha = \kappa \times \wp$. I assume that more policing yields a strictly higher detection chance, i.e., $\wp_e(e, \sigma) > 0$ for $e, \sigma > 0$.[10] Next, to capture that high severity crimes are less likely to avoid capture, I assume that the capture chance $\wp$ strictly rises in the severity of the offense: $\wp_\sigma(e, \sigma) > 0$ for $e, \sigma > 0$. Finally, to ensure well-behaved optimization problems for law enforcers and potential criminals, respectively, I assume that $\wp$ is concave in policing e (i.e., $\wp_{ee} \leq 0$) but convex in severity σ (i.e, $\wp_{\sigma\sigma} \geq 0$).

Since law enforcers and potential criminals make decisions at the margin, it is necessary to impose further restrictions on the capture chance $\wp(e, \sigma)$. The baseline case assumes that the marginal efficacy of enforcement is greater for high severity crimes. In other words, the *capture chance* $\wp(e, \sigma)$ *is supermodular in* (e, σ), or $\wp_{e\sigma} \geq 0$.[11] This assumption disci-

[10] For any differentiable real valued function $x \mapsto h$ on $\mathbb{R}^n$, I define $h_{x_i}(x) \equiv \partial h(x)/\partial x_i$.

[11] A real valued function $x \mapsto h$ on a lattice $X \subseteq \mathbb{R}^n$ is *supermodular (submodular)* if $h(\max\{x, x'\}) + h(\min\{x, x'\}) \geq (\leq) h(x) + h(x')$. When h is twice differentiable, then h is supermodular (submodular) iff $h_{x_i x_j}(x) \geq (\leq) 0$ for all $i \neq j$, by Topkis (1978). These definitions are strict if the inequalities are strict.

plines the non-trivial feedback effects between enforcers and criminals, while ensuring the equilibrium—outlined at the end of this section—is unique. Section §7.3 examines the submodular case, showing that the essence of the framework remains unchanged.

PAYOFFS. I begin with the behavior of potential criminals. An offense of severity σ yields a net *reward* $r(\sigma)$ to the criminal, where $r : [0,1] \to \mathbb{R}_+$ is twice continuously differentiable, strictly increasing, and concave: $r'(\sigma) > 0 > r''(\sigma)$ for all $\sigma > 0$, with $r(0) = 0$ and $\lim_{\sigma \downarrow 0} r'(\sigma) = \infty$. If a criminal is caught, they face a fixed *punishment* $f > 0$.[12] Thus, given enforcement e, a criminal chooses the severity σ of their offense to maximize expected *criminal profits*:

$$\Pi(\sigma, e) := r(\sigma) - \wp(e, \sigma) f. \tag{1}$$

As a result, a potential criminal with outside option ω finds it optimal to commit a crime if and only if, $\max_{\sigma \in [0,1]} \Pi(\sigma, e) \geq \omega$. When the support of ω is bounded (i.e., $\bar{w} < \infty$), I will assume that outside options are sufficiently spread, i.e. $\bar{w} \geq \max_{\sigma \in [0,1]} \Pi(\sigma, 0)$, ensuring that even in the absence of policing, some potential criminals prefer not to engage in crime.

I now turn to the payoffs of law enforcers. Enforcing the law is naturally costly. The *enforcement cost* function $c : [0,1] \to \mathbb{R}_+$ is assumed to be twice continuously differentiable, strictly increasing, and strictly convex: $c'(e), c''(e) > 0$ for $e > 0$, with $c(0) = c'(0) = 0$. As previously explained, a law enforcer encounters criminals at rate κ and, conditional on this event, successfully apprehends the offender with probability $\wp(e, \sigma)$. Therefore, given a crime rate κ and offense severity σ, a law enforcer chooses effort e to maximize arrest gains net of enforcement costs:

$$V(e, \sigma, \kappa) := \kappa \wp(e, \sigma) B - c(e), \tag{2}$$

where $B > 0$ denotes the *enforcement payoff* per criminal apprehended. This formulation reflects an *arrest-maximizing* objective rather than a crime-minimization goal. As discussed in the introduction, this behavior is supported by growing empirical evidence on police incentives (e.g., Stashko, 2022). It also arises naturally in this decentralized framework, where enforcers are modeled as "small" agents—each taking the crime rate as given and exerting discretionary effort to maximize arrests. In Section §7.1, I examine the polar case of centralized enforcement, where enforcers can internalize the effect of their joint actions on the aggregate crime rate in order to maximize total payoffs (2).

EQUILIBRIUM. In equilibrium, potential criminals and enforcers optimize independently and simultaneously. Formally, an *equilibrium* is a 5-tuple $(e^*, \sigma^*, \bar{\omega}^*, \kappa^*, \alpha^*)$ such that:

(i) Given enforcement e^*, severity σ^* maximizes criminal profits (1).

[12] Section §7.2 examines the case in which punishment f depends on the offense severity σ.

(ii) Given severity σ^* and crime rate κ^*, enforcement e^* maximizes (2).

(iii) Given (σ^*, e^*), marginal criminal $\bar{\omega}^*$ makes zero profits, i.e. $\Pi(\sigma^*, e^*) = \bar{\omega}^*$, while potential criminals with low outside options $\omega \leq \bar{\omega}^*$ commit crimes.

(iv) Given $(e^*, \sigma^*, \bar{\omega}^*)$, the crime rate is $\kappa^* = G(\bar{\omega}^*)$, and the arrest rate $\alpha^* = \wp(e^*, \sigma^*)\kappa^*$.

4 Equilibrium Analysis

THE OPTIMAL SEVERITY LOCUS $\mathcal{OS}$. I begin by characterizing the optimal crime severity. Observe that if maximal severity $\sigma = 1$ were optimal even under maximal enforcement $e = 1$, then it would remain optimal for all lower levels of enforcement, thereby trivializing the analysis. To ensure that criminals eventually face a non-trivial trade-off when choosing offense severity, a necessary condition on the primitives is the following:

$$r'(1) < \wp_\sigma(1, 1)f. \tag{3}$$

This condition effectively requires punishment f to be sufficiently high, $f > r'(1)/\wp_\sigma(1, 1)$, thereby ensuring that—at the margin—the expected legal cost outweighs the benefits when enforcement e is high enough. I henceforth focus on settings for which inequality (3) holds.

Let $\underline{e} := \inf\{e \in [0, 1] : r'(1) \leq \wp_\sigma(e, 1)f\}$.[13] This enforcement cutoff represents the lowest enforcement level such that any $e > \underline{e}$ would induce criminals to choose non-maximal severity, $\sigma < 1$. In other words, the optimal crime severity is maximal for all enforcement levels $e \in [0, \underline{e}]$; otherwise, it is non-maximal and characterized by the first-order condition:[14]

$$\Pi_\sigma(\sigma, e) = r'(\sigma) - \wp_\sigma(e, \sigma)f = 0. \tag{4}$$

At any interior maximum, the marginal expected punishment equals the marginal returns from crime. That is, when policing is not too low, a criminal finds it optimal to reduce their offense severity in order to decrease the chance of being apprehended. Thus, in addition to traditional deterrence effects that operate via criminals' extensive margin, here policing also has intensive effects by discouraging the "size" of the offense.

Now, because the capture chance $\wp$ is supermodular, the criminal profit function $\Pi(\sigma, e)$ (see (1)) is submodular in (e, σ). Indeed, differentiating Π_σ from (4) in enforcement level e yields:

$$\Pi_{\sigma e}(\sigma, e) = -\wp_{\sigma e}(e, \sigma)f \leq 0. \tag{5}$$

[13] Since $e \mapsto \wp_\sigma(e, 1)$ is continuous and strictly increasing, $\underline{e}$ is given by the solution to $r'(1) = \wp_\sigma(e, 1)$ whenever it exists on $[0, 1]$.

[14] For any enforcement level $e \geq 0$, the optimal crime severity is strictly positive, as $\lim_{\sigma \downarrow 0} r'(\sigma) = \infty$.

So, by Topkis' theorem (Topkis, 1978), the *optimal crime severity* $\Sigma(e) := \arg\max_{\sigma\in[0,1]} \Pi(\sigma, e)$ falls in enforcement level e. In other words, enforcement e and crime severity σ are *strategic substitutes for criminals* (Bulow et al., 1985): greater enforcement lowers the marginal profitability of high severity crimes. Finally, the Implicit Function Theorem ensures that $\Sigma : [0, 1] \to [0, 1]$ is continuous and differentiable almost everywhere. The $\mathcal{OS}$ locus depicts all pairs (σ, e) for which severity σ is optimal given e, as seen in the right panel of Figure 1.

THE SUPPLY OF CRIME. I now examine the potential criminals' extensive margin. Given enforcement $e \geq 0$, a potential criminal who commits a crime can get a payoff no greater than $\bar{\omega}(e)$, where

$$\bar{\omega}(e) := \max_{\sigma\geq 0} r(\sigma) - \wp(e, \sigma)f. \tag{6}$$

By the Envelope Theorem, $\bar{\omega}(\cdot)$ falls in e at rate $\bar{\omega}'(e) = -\wp_e(e, \Sigma(e))f < 0$. Thus, profits from crime are the lowest when there is full enforcement. Without loss of generality, I assume that $\bar{\omega}(1) \geq 0$.[15] Consequently, $\bar{\omega}(e)$ effectively characterizes the *marginal criminal* who is indifferent between engaging in crime and taking the outside option, given enforcement e. It follows that potential criminals with outside options $\omega \leq \bar{\omega}(e)$ enter the market, giving rise to *supply of crime*,

$$\mathcal{K}^S(e) := G(\bar{\omega}(e)), \tag{7}$$

which naturally falls as the enforcement level e rises.

THE INDUCED "DEMAND" FOR CRIME. I now study the law enforcers' optimization problem. Faced with a crime rate $\kappa > 0$ of severity $\sigma > 0$ crimes, a law enforcer maximizes (2). Notice that, at the optimum, enforcement must be strictly positive, since $c'(0) = 0$ and $\wp(\cdot, \sigma)$ is strictly increasing and concave; thus, $V(e, \sigma, \kappa)$ must strictly increase in e near $e = 0$. In addition, if the crime rate is not too high, the optimal enforcement is non-maximal, and thus it solves the first-order condition:

$$V_e(e, \sigma, \kappa) = \kappa\wp_e(e, \sigma)B - c'(e) = 0. \tag{8}$$

Let $\mathcal{E}(\kappa, \sigma) := \arg\max_{e\in[0,1]} V(e, \sigma, \kappa)$ denote the *optimal enforcement level*, given κ and σ. Notice that the payoff function $V(\cdot)$ in (2) is supermodular in $(e, (\sigma, \kappa))$. Indeed,

$$V_{e\sigma}(e, \sigma, \kappa) = \kappa\wp_{e\sigma}(e, \sigma)B \geq 0 \qquad \text{and} \qquad V_{e\kappa}(e, \sigma, \kappa) = \wp_e(e, \sigma)B > 0$$

Hence, by Topkis (1978), the optimal enforcement $\mathcal{E}(\kappa, \sigma)$ rises in (κ, σ), in that if $\sigma' \geq \sigma$

[15] Otherwise, there would exist a cutoff $\bar{e} < 1$ such that it is unprofitable to engage in crime provided $e \geq \bar{e}$, leading to zero crime. But this cutoff is irrelevant as zero crime can never be an equilibrium outcome, since law enforcers would find it suboptimal to keep enforcement high given their incentives in (2).

and $\kappa' \geq \kappa$, then $\mathcal{E}(\kappa', \sigma') \geq \mathcal{E}(\kappa, \sigma)$. In particular, unlike criminals, for a fixed crime rate κ, effort e and severity σ are *strategic complements for law enforcers*. Moreover, $\mathcal{E}(\cdot, \sigma) \equiv 1$ when the crime rate is high enough, namely, $\kappa \geq \bar{\kappa} \equiv c'(1)/[\wp_e(1, \sigma)B]$.

To construct a familiar way to perform comparative statics, notice that the enforcers' optimization gives rise to a "demand" for crime: How much crime κ makes enforcement e optimal for law enforcers? Since $\mathcal{E}(\cdot, \sigma)$ is strictly increasing on $[0, \bar{\kappa}]$, its inverse $\mathcal{K}^D(\cdot; \sigma) : [0, 1] \to [0, \bar{\kappa}]$ is well-defined, reflecting a *derived demand for crime*:

$$\mathcal{K}^D(e; \sigma) := c'(e)/[\wp_e(e, \sigma)B]. \tag{9}$$

That is, enforcement e is optimal provided the crime rate $\kappa = \mathcal{K}^D(e; \sigma)$. In the left panel of Figure 1, *the derive demand* $\mathcal{K}^D(\cdot; \sigma)$ slopes up in e and shifts left in the severity of the offense σ. Intuitively, for any crime rate $\kappa > 0$, more severe offenses elicit more enforcement.

The market clearing locus $\mathcal{MC}$. Equipped with "demand and supply" curves, I introduce the market clearing locus, which describes all pairs (e, σ) for which the market "clears," namely, $e = \mathcal{E}(\mathcal{K}^S(e), \sigma)$. When crime severity is not too high, the latter condition can be expressed as $\mathcal{K}^D(e; \sigma) = \mathcal{K}^S(e)$, namely, demand equals supply. In general, for each $\sigma \in [0, 1]$, the *market clearing enforcement* $e_{\mathcal{C}} : [0, 1] \to [0, 1]$ is given by:

$$e_{\mathcal{C}}(\sigma) := \sup \left\{ e \in [0, 1] : \mathcal{K}^S(e) \geq \mathcal{K}^D(e; \sigma) \right\}. \tag{10}$$

Lemma 1 shows that, by the continuity and monotonicity of the supply and demand curves, for each crime severity $\sigma > 0$ there exists a unique enforcement level e that "clears" the market. Two observations are in order at this point.

First, depending on primitives, there could exist a unique severity level $\bar{\sigma}$ such that the market clearing enforcement is maximal for all $\sigma \geq \bar{\sigma}$. This value is defined as the unique solution to $\mathcal{K}^S(1) = \mathcal{K}^D(1; \bar{\sigma})$. As $\mathcal{K}^D$ decreases in σ, it follows that $\mathcal{K}^S(1) > \mathcal{K}^D(1, \sigma)$ for all $\sigma > \bar{\sigma}$, and thus $e_{\mathcal{C}}(\sigma) \equiv 1$ on $[\bar{\sigma}, 1]$—explaining the flat region depicted in Figure 1 (middle).

Second, the market clearing locus $\mathcal{MC}$ must be upward sloping in (σ, e)-space. To see this, consider Figure 1 (left panel). There, a rise in severity σ shifts $\mathcal{K}^D$ left, leaving $\mathcal{K}^S$ unaffected. Hence, the market clearing enforcement rises along $\mathcal{K}^S$, indicating that pairs (σ, e) co-move along $\mathcal{MC}$, namely, the map $\sigma \mapsto e_{\mathcal{C}}(\sigma)$ is increasing in σ.

Equilibrium. The behavior of all agents is effectively projected into a graphical framework in the space of severity and enforcement levels (σ, e), in which any equilibrium is characterized by the optimal severity $\mathcal{OS}$ and market clearing $\mathcal{MC}$ loci. In equilibrium, criminal severity σ^* must be a best-reply to enforcement e^*, while enforcement level e^* must clear the market

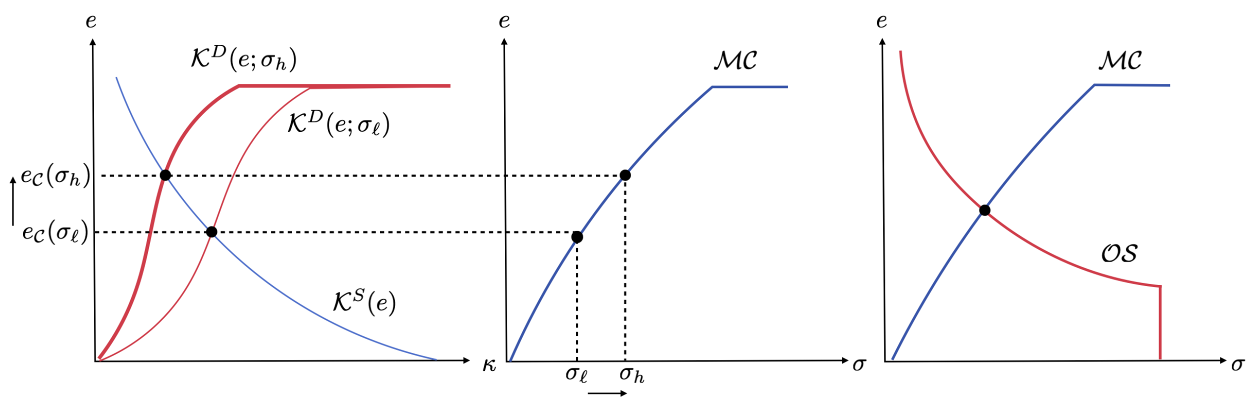

Figure 1: The left panel shows the crime supply curve $\mathcal{K}^S(e)$, which slopes downward in enforcement e, and the induced crime demand $\mathcal{K}^D(e;\sigma)$, which slopes upward in e for a given offense severity σ. As σ increases from σ_ℓ to σ_h, the demand curve shifts upward, resulting in a higher market-clearing enforcement level. The middle panel depicts the market-clearing locus $\mathcal{MC}$, which maps each offense severity σ to the enforcement level e that clears the market. The right panel depicts $\mathcal{MC}$ with the optimal severity locus $\mathcal{OS}$, which maps enforcement levels to criminals' optimal severity choices, illustrating the existence and uniqueness of equilibrium as the intersection of these two curves.

given severity σ^*. Thus, the intersections of $\mathcal{OS}$ and $\mathcal{MC}$ characterize all equilibria in the game. Moreover, by the respective slopes of $\mathcal{OS}$ and $\mathcal{MC}$, there is a unique intersection (σ^*, e^*), thereby proving the existence and uniqueness of an equilibrium; see Figure 1 (right).

Proposition 1 *There exists a unique equilibrium* $(e^*, \sigma^*, \kappa^*, \alpha^*)$.

Section §5 presents a fully solved numerical example illustrating how equilibrium outcomes vary with the model's primitives. Figure 2 depicts numerical simulations, showing three distinct equilibrium configurations. In the top-left panel, the equilibrium features maximal enforcement and low crime severity. The bottom left panel shows the opposite case. In the right panels, both enforcement and severity lie strictly between zero and one. The example shows how the nature of equilibrium—whether it involves maximal severity, full enforcement, or interior values—depends on the punishment level f and the enforcement payoff B. In particular, Figure 3 shows that an equilibrium with low enforcement ($e^* \leq \underline{e}$) and maximal severity ($\sigma^* = 1$) can arise when B is sufficiently small and f lies in an intermediate range—not too low to be ineffective, but not so high as to strongly deter crime. As explained in the next section, this could be more effectively counteracted with an strengthening of enforcement incentives (B) rather than an increase in punishment (f).

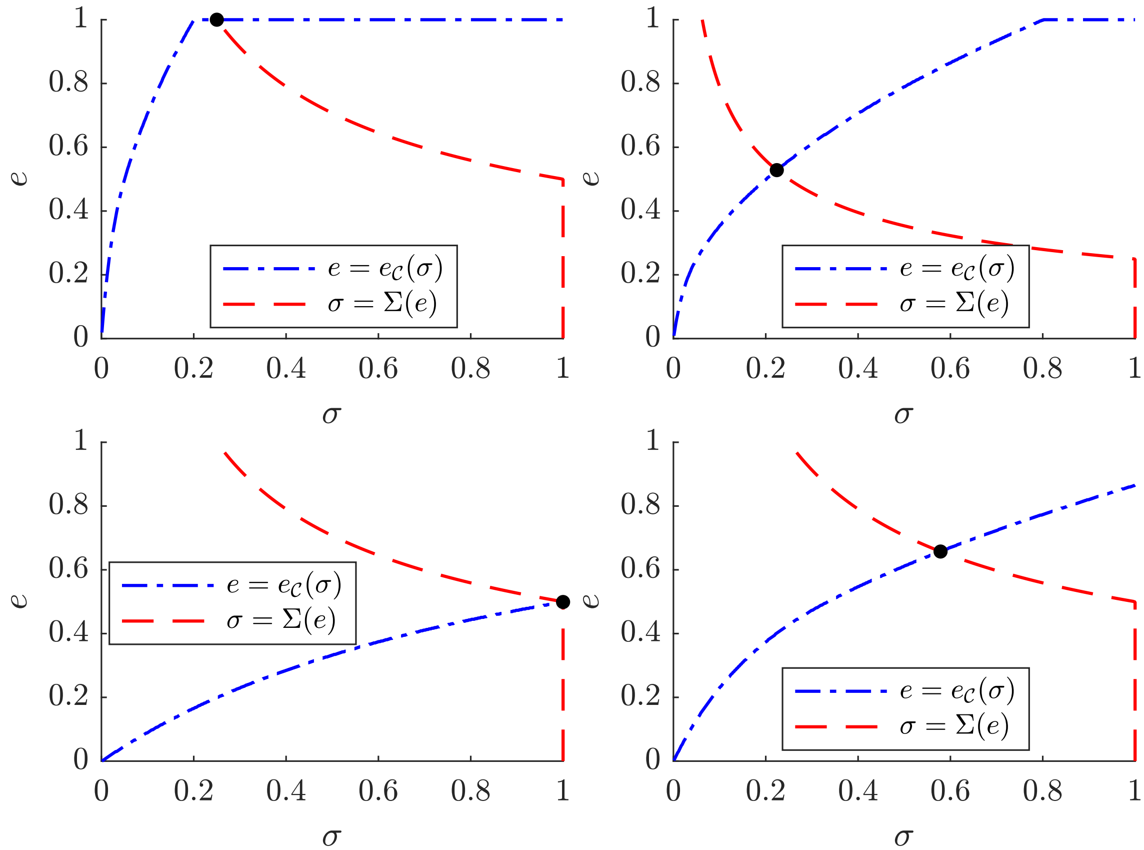

Figure 2: All panels use the following functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\wp(e, \sigma) = e\sigma$; and $G(\omega) = \omega$. Parameter values vary across panels as follows. Top left: $(B, f) = (20, 1)$; top right: $(B, f) = (10, 2)$; bottom left: $(B, f) = (1, 1)$; bottom right: $(B, f) = (3, 1)$.

5 An Illustrative Example

Consider enforcement costs $c(e) = e^2/2$; criminal rewards $r(\sigma) = \sqrt{\sigma}$; capture chance $\wp(e, \sigma) = e\sigma$; and uniformly distributed outside options on $[0, 1]$, i.e., $G(\omega) = \omega$. To satisfy condition (3), penalties are assumed to be high enough, or $f > 1/2$.

First, I find the optimal severity locus $\mathcal{OS}$. Given enforcement e, a criminal chooses the severity of their offense σ to maximize profits (1). In this example, $\underline{e}$ solves $r'(1) = \wp_\sigma(\underline{e}, 1)f$, namely, $\underline{e} = 1/(2f) \in (0, 1)$. Thus, the optimal crime severity is maximal, i.e., $\Sigma(e) \equiv 1$ for all $e \in [0, \underline{e}]$; otherwise, $\Sigma(e) \in (0, 1)$ solves the first-order condition (4), or:

$$\Sigma(e) \equiv \left(\frac{1}{2ef}\right)^2, \quad \forall e \in (\underline{e}, 1].$$

Second, I find the supply curve. Recall that the marginal criminal $\bar{\omega}(e) \equiv \Pi(\Sigma(e), e)$. Thus, $\bar{\omega}(e) \equiv 1 - ef$ for $e \in [0, \underline{e}]$, and $\bar{\omega}(e) \equiv 1/(4ef)$ for $e \in (\underline{e}, 1]$. Since outside options are uniformly distributed on $[0,1]$, the "supply of crime" is simply $\mathcal{K}^S(e) \equiv G(\bar{\omega}(e)) = \bar{\omega}(e)$:

$$\mathcal{K}^S(e) = \begin{cases} 1 - ef, & \text{if} \quad e \in [0, \underline{e}]; \\ \frac{1}{4ef}, & \text{if} \quad e \in (\underline{e}, 1]. \end{cases}$$

Third, I turn to the law enforcement side. Given crime severity σ and crime rate κ, a law enforcer chooses $e \in [0,1]$ to maximize (2). As outlined in §4, the optimal enforcement $\mathcal{E}(\kappa, \sigma) \equiv 1$ for $\kappa \geq \bar{\kappa} = 1/(\sigma B)$; otherwise, $\mathcal{E}(\kappa, \sigma) < 1$ and thus given by the first-order condition (8), namely, $\mathcal{E}(\kappa, \sigma) \equiv \kappa \sigma B$. The derived demand for crime is then:

$$\mathcal{K}^D(e; \sigma) = \frac{e}{\sigma B}.$$

Following §4, the market clearing locus $\mathcal{MC}$ consists of all pairs (σ, e) such that e "clears" the market given σ, namely, $e = e_{\mathcal{C}}(\sigma)$, where $e_{\mathcal{C}}(\cdot)$ is given by (10).

Finally, an equilibrium is generated by a tuple (σ^*, e^*) such that $e^* = e_{\mathcal{C}}(\sigma^*)$ and $\sigma^* = \Sigma(e^*)$. I next show that the properties of the equilibrium depend on both the incentives of law enforcers and potential criminals, captured by the parameters B and f, respectively. To this end, let $\bar{\sigma} = 4f/B \in \mathbb{R}_+$. There are two cases to consider.

- **Case 1:** $\bar{\sigma} \leq 1$, or $4f \leq B$. Then $\mathcal{K}^S(1) = \mathcal{K}^D(1; \bar{\sigma})$, and so $e_{\mathcal{C}}(\sigma) \equiv 1$ for all $\sigma \in [\bar{\sigma}, 1]$. Otherwise, for each $\sigma < \bar{\sigma}$, it follows that $\mathcal{K}^S(1) < \mathcal{K}^D(1, \sigma)$, and thus $e_{\mathcal{C}}(\sigma)$ equals the enforcement level $e \in (0,1)$ for which supply equals demand, $\mathcal{K}^S(e) = \mathcal{K}^D(e, \sigma)$. Consequently, the equilibrium could display either full or partial policing.

  Indeed, if $\bar{\sigma} \leq \Sigma(1)$, a condition that reduces to $B \geq 16f^3$, then the unique equilibrium entails $e^* = 1$ and $\sigma^* = \Sigma(1) = 1/(4f^2) < 1$, as seen in Figure 2 (top left panel). Otherwise, equilibrium enforcement $e^* > \underline{e}$ and so (e^*, σ^*) jointly solve $\mathcal{K}^D(e^*; \sigma^*) = \mathcal{K}^S(e^*)$ and $\sigma^* = \Sigma(e^*)$, yielding:

$$e^* = \left(\frac{B}{16f^3}\right)^{\frac{1}{4}} \in (0,1) \qquad \text{and} \qquad \sigma^* = \frac{1}{\sqrt{Bf}} \in (0,1). \tag{11}$$

  This equilibrium can be seen in the top right panel of Figure 2.

- **Case 2:** $\bar{\sigma} > 1$, or $4f > B$. Here, the market clearing locus obeys $e_{\mathcal{C}}(\sigma) < 1$ for all $\sigma \in [0,1]$. As in the previous case, two possible equilibria can emerge depending on primitives. In the bottom left panel of Figure 2, $e_{\mathcal{C}}(1) \leq \underline{e}$, or $Bf \leq 1$, and thus the equilibrium exhibits maximal crime severity $\sigma^* = 1$ and low enforcement $e^* \leq \underline{e}$, where

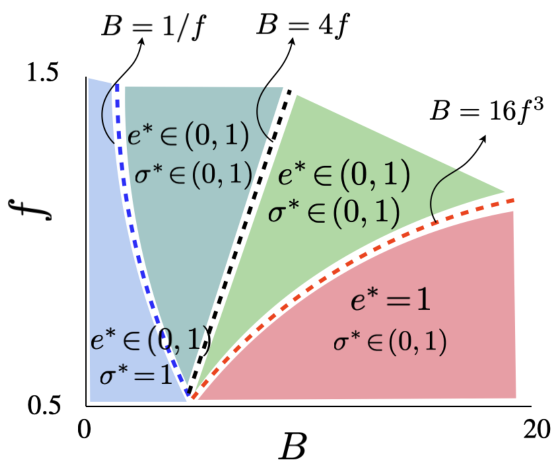

Figure 3: Functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\wp(e, \sigma) = e\sigma$; and $G(\omega) = \omega$.

$e^* = e_{\mathcal{C}}(1)$, i.e.:

$$e^* = \frac{B}{1 + Bf} \in (0, \underline{e}].$$

Conversely, if $Bf > 1$ so that $e_{\mathcal{C}}(1) > \underline{e}$, then $e^*, \sigma^* \in (0, 1)$ are given by (11), as depicted in the bottom right panel of Figure 2.

Figure 3 illustrates the regions of the (B, f) parameter space in which either enforcement effort or crime severity is maximal. Notably, achieving full enforcement does not necessarily require high enforcement benefits B. For example, an equilibrium with maximal policing and low crime severity can arise even when both punishment and enforcement incentives are relatively low—for instance, with $f \approx 1/2$ and $B \approx 2$. Conversely, deterring severe offenses (i.e., avoiding $\sigma^* = 1$) requires not only increasing punishment f but also strengthening enforcement incentives through a higher B. Indeed, if B is fixed and sufficiently small, increasing f alone may be insufficient to shift the equilibrium out of the region (shown in light blue) where severity remains maximal.

6 Comparative Statics

As illustrated in §5, equilibrium enforcement and crime severity are both strictly positive, though either may be maximal depending on the model's primitives. To streamline the analysis, I focus on *interior equilibria*, where $e^* \in (0, 1)$ and $\sigma^* \in (0, 1)$. This setting not only allows me to use the first-order conditions to conduct comparative statics, but also captures an environment in which equilibrium outcomes are sensitive to policy changes.

6.1 Harsher Punishment

Increasing penalties to deter crime is a policy intervention that directly impacts the supply side. As originally formalized by Becker (1968), when penalties become more severe, the expected cost of committing a crime rises, all else equal, leading potential offenders to abstain from criminal activity. However, a reduction in crime may, in turn, lead to a decline in enforcement efforts, since law enforcers allocate resources based on the prevalence of crime. In fact, a lower crime rate could reduce the expected benefits of maintaining high enforcement levels. If enforcement falls, some undeterred criminals may respond by committing more severe offenses, generating a feedback loop between enforcement and criminal behavior.

Motivated by this consideration, I examine how harsher penalties f influence equilibrium outcomes. I show that increasing punishment severity unambiguously reduces the equilibrium level of enforcement. However, it does not necessarily lead to a reduction in the severity of offenses. As recently discussed, as enforcement decreases, the marginal returns to committing more severe crimes increase—see equation (5). Proposition 2 provides clear sufficient conditions on the detection probability function $\wp(\cdot)$ under which higher penalties not only reduce the overall crime rate but also lead to a decline in offense severity.

Proposition 2 (Harsher Punishment) *If punishments are harsher (i.e., f rises), then the enforcement level e^* falls. The arrest rate α^* and the offense severity σ^* both fall, provided the detection chance $\wp(e, \sigma)$ is log-submodular in (e, σ). The crime rate κ^* falls if the detection chance $\wp(e, \sigma)$ is log-modular in $\wp(e, \sigma)$.*[16]

The intuition behind the proof is as follows. When penalties f increase, criminals are more deterred, reducing the overall crime supply—formally, $\mathcal{K}^S(e; f)$ shifts left, as seen in the right panel of Figure 4. Since enforcement is costly and not directly affected by penalties f, this decline in crime incentivizes law enforcers to scale back policing, shifting the market clearing $\mathcal{MC}$ locus down in the left panel of Figure 4. On the other hand, the optimal severity $\mathcal{OS}$ locus shifts left, as harsher penalties discourage severe offenses.

These forces create opposing effects on equilibrium crime severity. The net impact depends on a key primitive of the model: the detection probability function $\wp(e, \sigma)$. In particular, if the complementarity between enforcement effort e and crime severity σ in the detection process is not too strong—specifically, if $\wp$ is log-submodular—then an increase in punishment leads to fewer arrests, less severe offenses, and, under a stronger condition on $\wp$ (log-modularity), a lower crime rate. The technical details are provided in Appendix A.3.

[16] A positive function $h > 0$ is log-supermodular (log-submodular) if $\log(h)$ is supermodular (submodular). Hence, a log-modular function is both log-supermodular and log-submodular

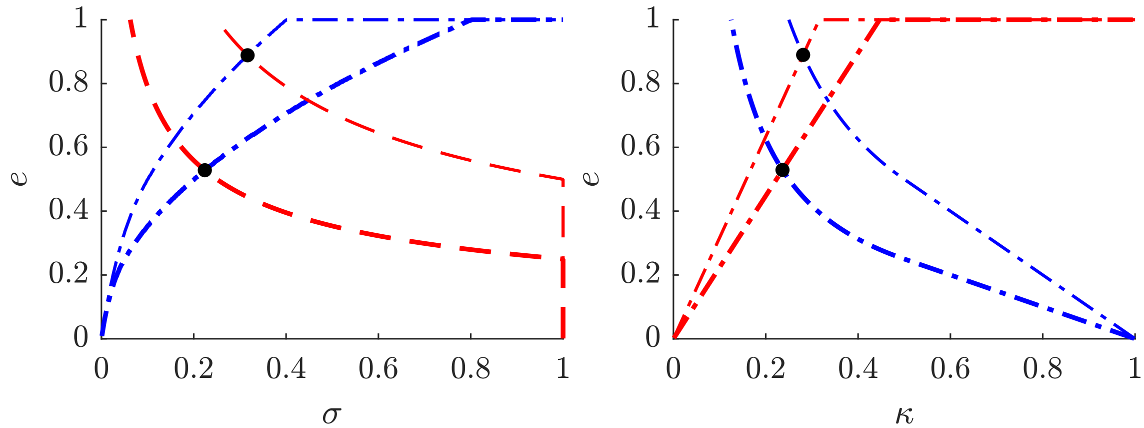

Figure 4: Functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\wp(e, \sigma) = e\sigma$; and $G(\omega) = \omega$. The figure depicts an increase in punishment f from $f = 1$ to $f = 2$. Left panel: The market clearing locus $\mathcal{MC}$ shifts left, while the optimal severity $\mathcal{OS}$ locus shifts right. Right panel: The supply curve $\mathcal{K}^S$ shifts left, while the demand curve $\mathcal{K}^D$ shifts right as a byproduct of lower severity offenses. The enforcement level, crime severity, and crime rate all fall.

From a technical perspective, this result underscores the role of complementarity in the detection probability function. On one hand, since $\wp(e, \sigma)$ is assumed to be supermodular, a reduction in enforcement following an increase in punishment incentivizes criminals to engage in more severe offenses. On the other hand, if this complementarity is not too strong—specifically, when $\wp$ is log-submodular[17]— the direct deterrent effect of harsher punishment dominates the indirect enforcement-driven effect, ultimately leading to a reduction in offense severity. Similarly, for the overall crime rate to decline, additional restrictions are necessary: while lower enforcement encourages more criminal entry, stricter punishments discourage it. To ensure that the crime rate does not rise—namely, for the direct deterrent effect to prevail—it is sufficient that the probability of detection is log-modular.

Proposition 2 echoes the point that discretionary enforcement can limit the effectiveness of policies aimed at reducing crime.[18] The proposition further reveals that the effectiveness of penalties hinges on how the probability of detection responds to both crime intensity and enforcement effort. When greater offense severity enhances policing efficiency (e.g., by generating more evidence) but not excessively so, then the reduction in enforcement caused by increased punishment will not fully offset the intended deterrent effect of the policy. Outside this world, the outcome relies on the model primitives in complex ways.

[17]If $\wp$ is log-submodular, then the cross-partial derivative $\wp_{e\sigma}$ is bounded; indeed, log-submodularity implies $\log(\wp)_{e\sigma} \leq 0$ or $(\wp_\sigma/\wp)_e \leq 0$, which is equivalent to $\wp_{e\sigma} \leq \wp_\sigma \wp_e/\wp$.

[18]Recently, Gonçalves and Mello (2023) empirically examined the role of police discretion in public safety, in a context in which police officers exercise discretion in assigning fines for traffic violations. They find that while stricter sanctions reduce future violations, discretionary enforcement can undermine public safety compared to a counterfactual setting without police discretion.

Remark 1 (Log-Modularity in Detection Probability) First, note that any detection technology with a "Cobb-Douglas" structure—where detection is multiplicative in enforcement and offense severity, such as $\wp(e, \sigma) = e^a \sigma^b$ with $a, b > 0$—satisfies log-modularity (and therefore log-submodularity). Hence, all the predictions of Proposition 2 apply in this case.

Second, the assumption that $\wp(e, \sigma)$ is log-modular is not merely a technical convenience. It can be grounded in a simple institutional mechanism in which detection occurs in two stages. First, citizens report offenses to law enforcement with probability $\rho(\sigma)$, increasing in offense severity. Second, conditional on a report, law enforcers exert effort e and successfully apprehend the offender with probability $\delta(e)$, increasing in e. The resulting detection probability is then given by $\wp(e, \sigma) = \delta(e) \cdot \rho(\sigma)$, which is naturally log-modular in (e, σ). This structure reflects observed features of real-world enforcement, where detection depends both on public cooperation and on the discretionary effort of law enforcers.[19]

6.2 Better Outside Options

Standard economic logic suggests that individuals weigh the benefits and costs of legal versus illegal activities, meaning that access to legitimate opportunities—such as employment, welfare support, or social insurance—can shape criminal behavior. When outside options deteriorate, individuals may turn to crime, as predicted by Becker (1968). But how does this shift affect the severity of their offenses and the corresponding enforcement response?

To answer this question, index the outside option distribution $G(\omega|\varphi)$, where $\varphi \in \mathbb{R}$. Say that *outside options improve* if $G(\cdot|\varphi_H) < G(\cdot|\varphi_L)$ when $\varphi_L < \varphi_H$, namely, $G(\cdot|\varphi_H)$ is better than $G(\cdot|\varphi_L)$ in the sense of First-Order Stochastic Dominance. I find that when outside options change, the amount of crime and the severity of offenses are negatively related.

Proposition 3 (Better Outside Options) *If outside options improve, then enforcement level e^* and crime rate κ^* both fall, while offense severity σ^* rises. The arrest rate α^* falls if the detection chance $\wp(e, \sigma)$ is log-supermodular in (e, σ).*

Proof: First, the supply of crime $\mathcal{K}^S(e|\varphi) \equiv G(\bar{\omega}(e)|\varphi)$ falls in φ for any enforcement e, while the demand for crime $\mathcal{K}^D$ is unaffected; hence, the market clearing enforcement falls along the demand $\mathcal{K}^D$. Thus, as seen in the left panel of Figure 5, the market clearing locus $\mathcal{MC}$

[19] According to the Bureau of Justice Statistics, reporting rates for violent crimes are significantly higher than for property crimes, suggesting that reporting likelihood increases with crime severity. See https://bjs.ojp.gov/library/publications/criminal-victimization-2023. See also Doleac and Sanders (2015), who examine the impact of ambient light on crime, leveraging Daylight Saving Time as a shock to visibility. They find that increased daylight during evening hours reduces robberies, underscoring the potential role of visibility and informal detection ("eyes on the street").

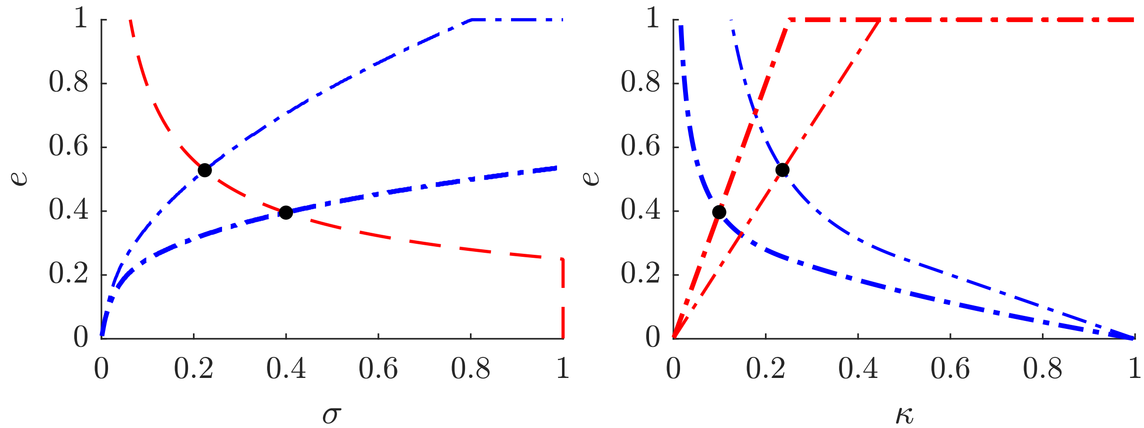

Figure 5: Functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\wp(e, \sigma) = e\sigma$; $B = 10$ and $f = 1$. The figure depicts an improvement in outside options, changing from $G(\omega) = \omega$ to $G(\omega) = \omega^2$. Left panel: The market clearing locus $\mathcal{MC}$ shifts right, while the optimal severity $\mathcal{OS}$ locus is unaffected. Right panel: The supply curve $\mathcal{K}^S$ shifts left, and does the demand curve $\mathcal{K}^D$ as crime severity rises. Overall, the crime rate falls but the offense severity rises.

shifts down in (σ, e)-space, and since the optimal severity locus $\mathcal{OS}$ is constant in φ, by (4), the equilibrium enforcement level e^* falls but the offense severity σ^* rises.

Now the crime rate falls due to two reinforcing effects: supply $\mathcal{K}(\cdot|\varphi)$ and demand $\mathcal{K}^D(\cdot, \sigma)$ both shift left (see Figure 5, right). Finally, using (9), the arrest rate is given by $\mathcal{A}(e, \sigma) := \wp(e, \sigma)\mathcal{K}^D(e, \sigma) = c'(e)\wp(e, \sigma)/(\wp_e(e, \sigma)B)$. Notice that, since $\wp_e > 0 \geq \wp_{ee}$, the ratio $\wp/\wp_e$ increases in e. Also, if $\wp(e, \sigma)$ is log-supermodular in (e, σ), then $\wp/\wp_e$ decreases in σ (as log-supermodularity implies $\wp_e/\wp$ is weakly increasing in σ). Thus, since $c'' \geq 0$, function $\mathcal{A}(e, \sigma)$ increases in e and decreases in σ. Altogether, since better outside options lead to less enforcement e^* and more severe offenses σ^*, the arrest rate $\alpha^* = \mathcal{A}(e^*, \sigma^*)$ must fall. □

Reversing the logic, Proposition 3 finds that worse outside options not only raise the number of individuals engaging in crime, but also decreases the severity of each offense. This result seems to be consistent with the empirical literature. For example, Bignon et al. (2015) documents the evolution of crime rates in France in 19th century, finding that from 1826-1936, the phylloxera crises destroyed 40% of France's vineyards—which can be viewed a negative income shock, or worse outside options—causing a substantial increase in property crime and a significant decrease in violent crime. More recently, Melander and Miotto (2023) examine the impact of the 1834 Poor Law Amendment Act on crime in England and Wales, analyzing how the reduction in welfare support affected criminal activity. They find that areas experiencing greater cuts in welfare spending saw a significant rise in non-violent property crimes such as larceny. Similarly, Giulietti and McConnell (2024) investigate the impact of the UK's Welfare Reform Act 2012 on crime, focusing on how austerity-driven welfare cuts influenced criminal behavior. They find that areas more exposed to the reforms experienced a significant rise in crime, driven primarily by new individuals engaging in criminal activity rather than existing offenders committing more crimes.[20]

6.3 Changing the Detection Technology

The probability of detection can be influenced by environmental or technological factors. For example, Vollaard (2017) examines illegal marine oil discharges and finds that ship operators strategically commit offenses at night when detection is less effective. From a technological perspective, Anker et al. (2021) analyze the expansion of Denmark's DNA database and show that increasing the likelihood of offender identification significantly reduces crime.

Motivated by these findings, this section examines how equilibrium outcomes change when the probability of detection varies. To this end, suppose the detection probability is indexed by a "technology" parameter $\varphi \in (0, 1]$. In particular, assume that the parametrized detection function $\tilde{\wp}(e, \sigma|\varphi)$ satisfies $\tilde{\wp}(e, \sigma|\varphi) \equiv \varphi\wp(e, \sigma)$, where $\wp(e, \sigma)$ satisfies all assumptions stated in Section §3, so that the baseline case is recovered by setting $\varphi = 1$. Notice that, an increase in technology φ not only raises the likelihood of detection ($\tilde{\wp}_\varphi > 0$), all else equal, but also enhances the effectiveness of policing at the margin ($\tilde{\wp}_{e\varphi} > 0$).

Unlike the previous exercises in this section, here a change in detection technology directly affects both sides of the market—potential criminals and law enforcers—complicating the comparative statics. I next show that, provided the supply of crime is sufficiently elastic in φ, both the equilibrium offense severity and the enforcement level decrease as detection technology improves. More precisely, let $\bar{\omega}(e|\varphi) := \max_{\sigma\in[0,1]} r(\sigma) - \tilde{\wp}(e, \sigma|\varphi)f$ denote the potential criminal who is indifferent between committing a crime and abstaining when policing is e and technology is φ. The supply curve $\mathcal{K}^S(e|\varphi) = G(\bar{\omega}(e|\varphi))$ is said to be *sufficiently elastic* in φ, given e, if

$$\left|\frac{\varphi\mathcal{K}^S_\varphi(e|\varphi)}{\mathcal{K}^S(e|\varphi)}\right| \geq 1,$$

which can be equivalently expressed as $(g(\bar{\omega})/G(\bar{\omega}))\tilde{\wp}f \geq 1$, using the envelope theorem.

Proposition 4 (Detection Technology) *Suppose that $(g(\bar{\omega})/G(\bar{\omega}))\tilde{\wp}f \geq 1$. If the detection technology improves (i.e., φ rises), then the enforcement level e^* falls. In addition, the offense severity σ^* and the arrest rate α^* both fall if $\tilde{\wp}$ is log-submodular in (e, σ). Finally,*

[20] Britto et al. (2022) examine the impact of job loss and unemployment insurance on crime in Brazil, using mass layoffs as an exogenous shock to employment. They find that displaced workers are more likely to engage in criminal activity, with property crime increasing proportionally more than violent crime.

the crime rate κ^* falls, if one further assumes that $\tilde{\wp}$ is log-modular in (e, σ).

Consider an increase in the detection technology parameter φ. First, this increases the likelihood that criminals are caught, leading to a reduction in both criminal entry and offense severity. As seen in Figure 6, both the optimal severity locus $\mathcal{OS}$ and the crime supply curve $\mathcal{K}^S$ shift left. Meanwhile, a higher probability of detection enhances the effectiveness of law enforcement, reflected by an upward shift in the demand curve $\mathcal{K}^D$. As a result, the overall impact on the market-clearing locus $\mathcal{MC}$ is ambiguous. Appendix A.4 shows that if the supply curve is sufficiently elastic with respect to the detection probability—formally, if $g(\bar{\omega})/G(\bar{\omega}) \cdot \tilde{\wp} f \geq 1$—then the supply-side effect dominates, causing $\mathcal{MC}$ to weakly decrease. In Figure 6 (left panel), the market clearing locus $\mathcal{MC}$ remains unchanged since the functional forms used render the supply to be unit elastic in φ, or $g(\bar{\omega})/G(\bar{\omega}) \cdot \tilde{\wp} f = 1$;[21] thus, the effects of the supply and demand shifts on the market clearing enforcement offset each other.

Since $\mathcal{OS}$ contracts while $\mathcal{MC}$ expands, the equilibrium enforcement level e^* unambiguously decreases. However, the effect on equilibrium severity σ^* is more subtle. Appendix A.4 demonstrates that if the probability of detection is log-submodular in (e, σ), then the $\mathcal{OS}$ locus shifts downward more than $\mathcal{MC}$, leading to a reduction in offense severity σ^*.

A few observations are in order. First, from a technical standpoint, Proposition 4 assumes that the supply of crime is sufficiently elastic. While this assumption depends on endogenous variables and could be verified ex post, it can also be empirically tested. For instance, Anker et al. (2021) finds that a 1% increase in the probability of detection leads to more than a 2% reduction in crime, consistent with a sufficiently elastic supply curve.

Second, from an economic perspective, the result highlights the complex feedback effects introduced by improvements in detection technology, showing that reductions in crime and offense severity are not necessarily guaranteed. However, the analysis also identifies a clear sufficient condition on the probability of detection under which technological advancements would lead to desirable outcomes, such as reductions in crime, arrests, and offense severity. These new predictions on crime severity and enforcement levels could be empirically tested, providing a way to refine or complement existing findings from the aforementioned studies.

Finally, from a policy viewpoint, Proposition 4 suggests that in environments where criminals are naturally harder to detect—such as at nighttime—one should expect higher

[21] Consider the example from Section §5, but with detection technology $\tilde{\wp}(e, \sigma|\varphi) \equiv \varphi e \sigma$, $\varphi \in (0, 1]$. Notice that in an interior equilibrium, the optimal severity is given by $\Sigma(e|\varphi) = (\frac{1}{2\varphi e f})^2$, and thus $\bar{\omega}(e|\varphi) = \tilde{\wp}(e, \Sigma|\varphi) f = 1/(4\varphi e f)$. Consequently,

$$\Big|\frac{\varphi \mathcal{K}^S_\varphi(e|\varphi)}{\mathcal{K}^S(e|\varphi)}\Big| = \frac{g(\bar{\omega})\tilde{\wp} f}{G(\bar{\omega})} = \frac{\tilde{\wp} f}{\bar{\omega}} = 1.$$

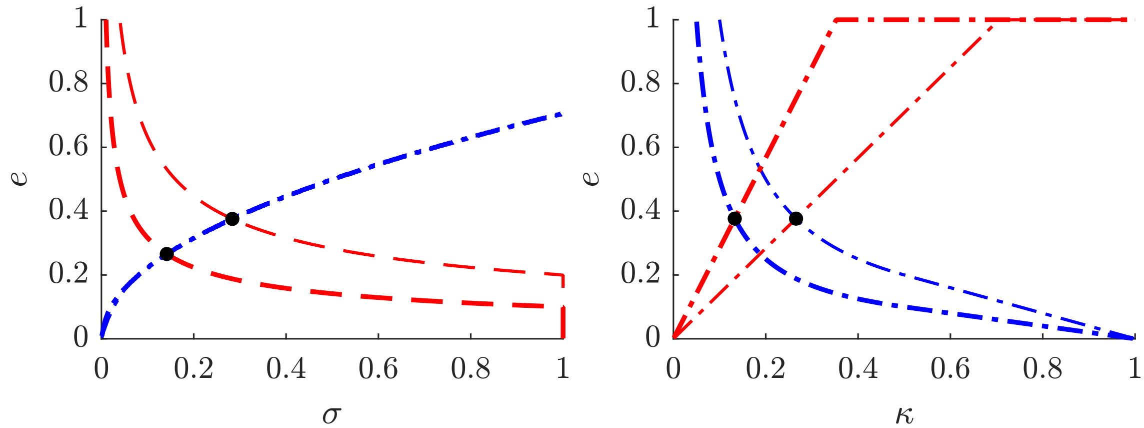

Figure 6: Functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\tilde{\wp}(e, \sigma|\varphi) = \varphi e \sigma$; $G(\omega) = \omega$; $B = 10$ and $f = 5$. The figure depicts an improvement in the detection probability, captured by varying φ from $\varphi = 0.5$ to $\varphi = 1$. Left panel: The optimal severity locus $\mathcal{OS}$ shifts left, while the market clearing locus $\mathcal{MC}$ is unchanged, as a consequence of the supply curve being unit elastic in φ. Right panel: The supply curve $\mathcal{K}^S$ and demand curve $\mathcal{K}^D$ shift left, leaving the market clearing enforcement unaffected. Overall, the enforcement level falls but so does the offense severity.

crime rates, increased enforcement activity, and more severe offenses compared to daytime. A natural policy implication is to modify the environmental conditions that shape the behavior of both criminals and law enforcers. Recently, Chalfin et al. (2022) examine the impact of improved street lighting on crime finding a more pronounced reduction in violent crimes compared to property crimes, without a corresponding increase in arrests. This empirical findings are consistent with Proposition 4 in that increasing the detection probability can deter crime and severity without eliciting greater enforcement efforts.

6.4 Strengthening Enforcement

Finally, I examine the equilibrium effects of incentivizing arrests on crime outcomes. In empirical research, arrests are often used as a proxy for police enforcement. However, as this paper demonstrates, the relationship between enforcement and arrests is more nuanced, as the arrest rate depends not only on law enforcement behavior but also on the level of crime. Consequently, an increase in arrests does not necessarily indicate greater enforcement, nor does a decrease in arrests imply less enforcement. While the previous comparative statics in this section revealed a positive association between enforcement and arrests, the next result highlights a case where these two variables need not move together in equilibrium. Specifically, I show that when law enforcers have stronger incentives to apprehend criminals—formally, when B increases—equilibrium enforcement rises, but the arrest rate falls.

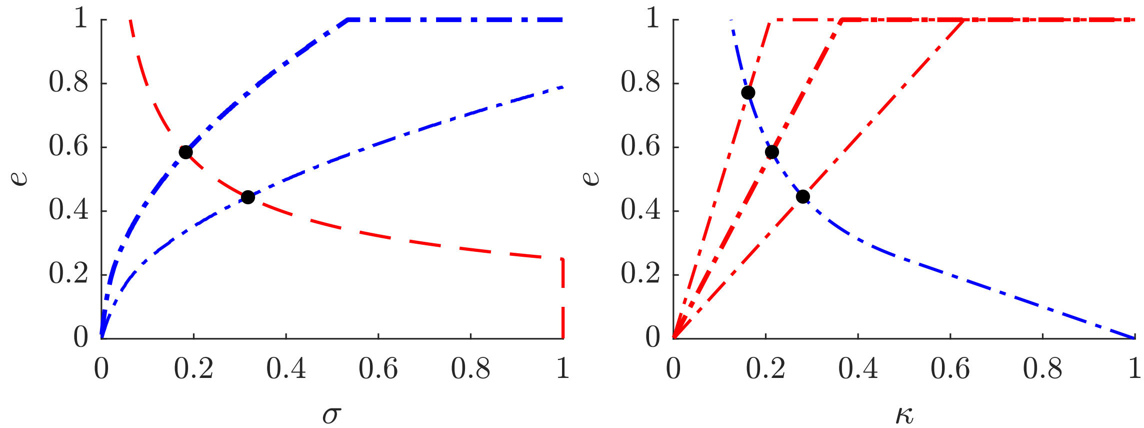

Figure 7: Functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\wp(e, \sigma) = e\sigma$; $G(\omega) = \omega$; and $f = 2$. The figure depicts an increase arrest incentives B, increasing from $B = 5$ to $B = 10$. Left panel: The market clearing locus $\mathcal{MC}$ shifts left, while the optimal severity $\mathcal{OS}$ locus is unchanged. Right panel: The supply curve $\mathcal{K}^S$ is unaffected, while the demand curve $\mathcal{K}^D$ shifts left but then it is partially offset as a result of lower offense severity. The enforcement level, crime severity, and crime rate all fall.

Proposition 5 (Greater Enforcement Payoff) *If the payoff per criminal apprehended B rises, then the enforcement level e^* rises, while the offense severity σ^* and crime rate κ^* fall. The arrest rate α^* falls if the detection chance $\wp$ and criminal rewards r satisfy:*

$$\frac{\wp_{\sigma\sigma}}{\wp_\sigma} - \frac{\wp_{\sigma e}}{\wp_e} - \frac{r''}{r'} \leq 0. \qquad (12)$$

This condition holds if $\wp$ is log-modular in (e, σ), and $\wp$ and r are isoelastic in σ.

Suppose that the enforcement payoff B increases. In the (κ, e)-space, the derived demand for crime $\mathcal{K}^D(e, \sigma) = c'(e)/[\wp_e(e, \sigma)B]$ shifts left, while the supply curve $\mathcal{K}^S(e)$ remains unchanged. Consequently, for a fixed offense severity σ, the market-clearing enforcement level must rise. In the (σ, e)-space, this translates into a leftward shift of the market-clearing locus $\mathcal{MC}$, as shown in the left panel of Figure 7. Because the optimal severity locus $\mathcal{OS}$ is unaffected by B, the equilibrium enforcement level e^* increases, while the corresponding offense severity σ^* decreases. As a result, the equilibrium crime rate $\kappa^* = G(\bar{\omega}(e^*))$ falls. The effect on the arrest rate $\alpha^* = \wp(e^*, \sigma^*)\kappa^*$, however, is more subtle. While κ^* falls with B, the change in α^* depends on how B influences the detection probability $\wp(e^*, \sigma^*)$. Appendix A.5 shows that under condition (12), the detection chance $\wp(e^*, \sigma^*)$ falls with B, implying that the arrest rate α^* must also fall.

Proposition 5 uncovers a counterintuitive yet important insight: increasing enforcement efforts can actually lead to fewer arrests. The underlying mechanism is that stronger incentives for law enforcers to apprehend criminals drive them to exert greater effort, which in turn deters individuals from both committing crimes and engaging in more severe offenses. As a result, potential offenders respond with fewer and less severe offenses, ultimately lowering the overall crime rate. Consequently, despite higher enforcement efforts, the total number of arrests decreases simply because fewer individuals are committing crimes in the first place.

7 Extensions

7.1 Centralized Enforcement

I now turn to the study of centralized enforcement, as analyzed in Lazear (2006), Eeckhout et al. (2010), and Gao and Vásquez (2024). These studies examine the effectiveness of committing to policing strategies in contexts where a single police agency is responsible for allocating resources.[22] In this literature, the police agency typically seeks to minimize total crime; however, some studies have also explored scenarios where the objective is to maximize total arrests (e.g., Persico, 2002). Regardless of the specific enforcement goal, the interaction between law enforcement and potential criminals takes on a sequential nature: the police agency first selects an enforcement level $e \geq 0$, and criminals then adjust their behavior in response. This structure highlights the strategic role of pre-committed enforcement policies in shaping criminal incentives and, ultimately, crime outcomes. I next show that the combination of centralized enforcement with arrest driven incentives can lead to undesirable outcomes. Specifically, rather than reducing crime, it can incentivize more offenses, each of greater severity, while simultaneously reducing overall enforcement effort.

Given enforcement level $e \geq 0$, if a potential criminal chooses to commit a crime, then the crime intensity must maximize criminal profits (1) and thus obey $\sigma = \Sigma(e)$. In turn, the mass of potential criminals who find it optimal to engage in crime, given e, must be given by $\kappa = \mathcal{K}^S(e)$. Thus, the police agency chooses enforcement level $e \in [0, 1]$ to maximize $\mathcal{V}(e) := V(e, \Sigma(e), \mathcal{K}^S(e))$, where $V(e, \sigma, \kappa)$ is given in (2). That is, the police agency solves:

$$\max_{e\in[0,1]} \underbrace{\mathcal{K}^S(e)\wp(e,\Sigma(e))B - c(e)}_{\mathcal{V}(e):=}.$$

Two observations are in order. First, the objective function is strictly increasing near $e = 0$, and thus the optimizer $e^C := \arg\max_{e\in[0,1]} \mathcal{V}(e)$ must be strictly positive. To see this, recall

[22] For instance, Eeckhout et al. (2010) provides empirical evidence of police agencies committing to targeted crackdowns to enhance deterrence.

that crime severity $\Sigma(e) \equiv 1$ for $e \in [0, \underline{e}]$, and thus $\Sigma'(e) = 0$ on $[0, \underline{e})$. Consequently,

$$\mathcal{V}'(0) = \mathcal{K}^S(0)\wp_e(0, 1)B > 0,$$

where the inequality holds since $\mathcal{K}^S(0) > 0$, and $\wp_e(0, 1) > 0$ as $\wp$ is strictly increasing in e.

Second, while the optimal centralized enforcement e^C is positive, it could exhibit very little enforcement compared to its decentralized counterpart, thereby increasing the marginal returns of high severity offenses. Let $(e^C, \sigma^C, \kappa^C, \alpha^C)$ denote the centralized equilibrium, and $(e^*, \sigma^*, \kappa^*, \alpha^*)$ denote its decentralized counterpart.

Proposition 6 *Suppose that the arrest rate $\mathcal{K}^S(e)\wp(e, \Sigma(e))$ is concave in enforcement e. If $e^* < 1$ then the centralized equilibrium entails less enforcement $e^C < e^*$, more severity per crime $\sigma^C > \sigma^*$, and more total crime $\kappa^C > \kappa^*$ than the decentralized counterpart. Also, if the detection chance $\wp$ and criminal rewards r satisfy (12), crime severity is maximal $\sigma^C = 1$.*

Proof: Notice that for almost all e,[23]

$$\mathcal{V}'(e) = \mathcal{K}^S(e)\left(\wp_e(e, \Sigma(e)) + \wp_\sigma(e, \Sigma(e))\frac{d\Sigma(e)}{de}\right)B + \mathcal{K}^S_e(e)\wp(e, \Sigma(e))B - c'(e)$$

Meanwhile, in the decentralized equilibrium, if $e^* < 1$ then it must, generically, satisfy

$$\mathcal{K}^S(e^*)\wp_e(e^*, \Sigma(e^*))B - c'(e^*) = 0,$$

and thus

$$\mathcal{V}'(e^*) = \mathcal{K}^S(e^*)\wp_\sigma(e^*, \Sigma(e^*))\frac{d\Sigma(e^*)}{de}B + \mathcal{K}^S_e(e^*)\wp(e, \Sigma(e^*))B < 0,$$

where the inequality follows as $\wp_\sigma > 0 \geq d\Sigma/de$ and $\mathcal{K}^S_e < 0$. Since $\mathcal{V}(e)$ is concave in e (as $\mathcal{K}^S(e)\wp(e, \Sigma(e))$ is concave and $c(e)$ is convex), it follows that $e_C < e^*$. Consequently, $\sigma_C = \Sigma(e^C) > \sigma^* = \Sigma(e^*)$, and $\kappa^C = \mathcal{K}^S(e^C) > \kappa^* = \mathcal{K}^S(e^C)$.

Now, suppose that $\wp$ and r jointly satisfy (12). Then, the proof of Proposition 5 shows that $e \mapsto \wp(e, \Sigma(e))$ is decreasing in $e > \underline{e}$, and since the crime rate $\mathcal{K}^S(e)$ is also strictly decreasing, it follows that total arrest $\mathcal{K}^S(e) \times \wp(e, \Sigma(e))$ is a strictly decreasing function. Therefore, $\max_{e \in [\underline{e}, 1]} \mathcal{V}(e) = \mathcal{V}(\underline{e})$, implying that e^C cannot be above $\underline{e}$; otherwise, one could decrease e to increase arrest benefits while decreasing enforcement costs. As a result, the optimizer $e^C \in (0, \underline{e}]$ and so the resulting severity is maximal $\sigma^C = \Sigma(e^C) = 1$. □

This result highlights a key institutional insight: while arrest driven incentives can be effective in reducing crime when enforcement is decentralized (Proposition 5), this need not

[23] $\mathcal{V}(e)$ is not differentiable at $e = \underline{e}$ due to $\Sigma(e)$ having a "kink" at $e = \underline{e}$.

extend to centralized settings. When enforcement is centralized and individual enforcers have limited discretion, rewarding arrests may distort incentives and lead to detrimental outcomes. In such contexts, a more appropriate objective for the police agency is not to maximize arrests, but to minimize crime directly. This distinction underscores the importance of aligning incentives with the institutional environment in which enforcement operates.

7.2 Convex Penalties

In this section, I demonstrate that the results of the paper extend to settings where penalties depend on the severity of the offense. To illustrate this, and with a slight abuse of notation, suppose that if a lawbreaker is caught, they must pay a *fine* $f(\sigma)$, where $f : [0,1] \to [0,\infty)$ is strictly increasing and convex, satisfying $f'(\sigma) > 0$ and $f''(\sigma) > 0$ for $\sigma > 0$, with $f(0) = f'(0) = 0$. These conditions ensure that the severity of the penalty increases with the severity of the offense at increasing rates. As a result, a criminal understands that increasing the severity of their offense not only raises the likelihood of apprehension but also leads to a harsher legal penalty if caught.

The criminal's expected profit function (1) now takes the form:

$$\Pi(\sigma, e) := r(\sigma) - \wp(e,\sigma) f(\sigma).$$

Two key observations arise from this formulation and criminal optimality.

First, the optimal severity choice, captured by the function $\Sigma(e) := \arg\max_\sigma \Pi(\sigma, e)$, remains downward sloping. This property is driven by the submodularity properties of Π. Specifically, for $e, \sigma \in (0,1)$, it follows:

$$\Pi_{\sigma e}(\sigma, e) = -\wp_{\sigma e}(e,\sigma) f(\sigma) - \wp_e(e,\sigma) f'(\sigma) < 0.$$

This implies that Π is submodular in (e, σ), ensuring that $\Sigma(e)$ decreases with e by Topkis' Theorem (Topkis, 1998). Furthermore, the substitution effect between enforcement and offense severity strengthens in this case. Even if $\wp_{e\sigma} = 0$, it is still the case that $\Pi_{\sigma e}(\sigma, e) < 0$, since the marginal cost of committing a more severe offense reflects not just the increased detection probability but also the possibility of a higher fine, $\wp \times f'(\sigma)$. However, enforcement may have limited deterrent effect at low levels: if condition (3) is adjusted so that $r'(1) < \wp_\sigma(1,1) f(1) - \wp(1,1) f'(1)$, then a threshold $\underline{e} \in (0,1)$ would exist such that severity is maximal, namely, $\Sigma(e) \equiv 1$ for all $e \in [0, \underline{e}]$. Beyond this threshold, $\Sigma(e)$ is determined by the first-order condition, $\Pi_\sigma(\Sigma(e), e) \equiv 0$, as in the baseline model.

The second key observation is that the supply of crime retains the core properties established in the baseline model. The marginal criminal's outside option is given by $\bar{\omega}(e) := \max_\sigma \Pi(\sigma, e)$. By the Envelope Theorem,

$$\bar{\omega}'(e) = -\wp_e(e, \Sigma(e)) f(\Sigma(e)) < 0,$$

ensuring that the crime supply function $\mathcal{K}^S(e) = G(\bar{\omega}(e))$ remains downward sloping.

However, the comparative statics of penalties (*cf.* Proposition 2 in §6.1) is now more complicated, as one must clearly specify the meaning of "harsher penalties." To formalize this, I introduce a parameterized family of fine function $f(\sigma|\varphi)$, where $\varphi \in \mathbb{R}$ indexes penalty severity and satisfies $f_\varphi(\sigma|\varphi) > 0$. Appendix B establishes that Proposition 2 mostly generalizes under these conditions, provided $f(\sigma|\varphi)$ is log-supermodular in (σ, φ)—meaning that a rise in φ increases the elasticity of fines with respect to offense severity (Proposition B.1). A simple example that satisfies these properties is: $f(\sigma|\varphi) = (1+\sigma)^\varphi$ with $\varphi \geq 1$.

7.3 Submodular Detection Probability

In the baseline case, the detection probability $\wp(e, \sigma)$ is assumed to be supermodular in (e, σ). This section examines the opposite case, where $\wp(e, \sigma)$ is *submodular* in (e, σ). This reflects scenarios in which more severe offenses *diminish* the marginal effectiveness of greater enforcement.[24] Formally, I assume that $\wp(e, \sigma)$ retains the same key properties from the baseline model—namely, that it is increasing and concave in e and increasing and convex in σ—except that now it is submodular in (e, σ), meaning that $\wp_{e\sigma} \leq 0$. A simple example satisfying these conditions is $\wp(e, \sigma) = e + \sigma - e\sigma$.

This assumption carries important implications. First, observe that criminal profits $\Pi(\sigma, e)$ in (1) are now supermodular in (e, σ) (*cf.* (5)). As a result, the optimal severity function $\Sigma(e) = \arg\max_\sigma \Pi(\sigma, e)$ is now *increasing* in e. This means that greater enforcement incentivizes criminals to commit more severe offenses—a stark contrast to the baseline case. However, despite this shift in offense severity, the crime supply curve $\mathcal{K}^S(e)$ (see (7)) remains downward sloping. Indeed, differentiating $\mathcal{K}$ yields:

$$\frac{d\mathcal{K}^S(e)}{de} = g(\bar{\omega}(e))\bar{\omega}'(e),$$

where the marginal criminal $\bar{\omega}(e)$ (see (6)) satisfies $\bar{\omega}'(e) = -\wp_e(e, \Sigma(e)) f < 0$. Thus, while higher enforcement reduces the overall crime rate, it also leads to more severe offenses among

[24]The severity of an offense can reflect its level of sophistication. In cybercrime, minor offenses (e.g., phishing scams) are easily detected using standard tools (e.g., IP tracking), while more advanced crimes (e.g., nation-state hacking, cryptocurrency laundering) leverage encryption and decentralized networks (Cong et al., 2025), reducing the marginal effectiveness of additional enforcement.

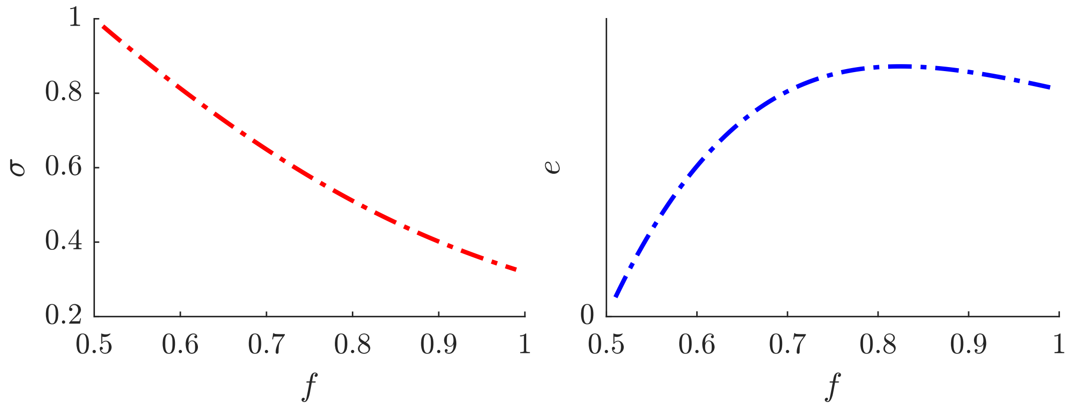

Figure 8: Functional forms: $c(e) = e^2/2$; $r(\sigma) = \sqrt{\sigma}$; $\wp(e,\sigma) = e + \sigma - e\sigma$; $G(\omega) = \omega$; and $B = 1$. The figure depicts how the equilibrium crime severity σ^* (left panel) and the equilibrium enforcement intensity e^* (right panel) vary as punishment increases $f \in [0.5, 1]$.

those who still choose to commit crimes.

For law enforcers, the derived demand for crime $\mathcal{K}^D(e,\sigma)$ (9) remains increasing in e, ensuring that the market-clearing enforcement level $e_{\mathcal{C}}(\sigma)$ (see (10)) is well-defined. However, a critical departure from the baseline case is that *higher offense severity increases the demand for crime*, implying that the market clearing enforcement level $e_{\mathcal{C}}(\sigma)$ must decrease in σ.

These changes lead to a key observation: the submodularity of $\wp$ reverses the monotonicity of both the optimal severity locus ($\mathcal{OS}$) and the market-clearing locus ($\mathcal{MC}$). Nevertheless, since these two loci move in opposite directions, their intersection remains unique—ensuring that a unique equilibrium exists, just as in the baseline model.

A natural question is: how do harsher punishments affect crime and enforcement in this setting? Several key observations emerge. First, following the same logic as in the proof of Proposition 2, the optimal severity function $\Sigma(e; f)$ decreases with f for each fixed e. Second, the crime supply function $\mathcal{K}^S(e; f)$ shifts leftward, while the crime demand function $\mathcal{K}^D$ remains unchanged, implying that the market-clearing enforcement level $e_{\mathcal{C}}(\sigma; f)$ must also fall for each σ. Crucially, this means that—unlike in §6.1—both the $\mathcal{OS}$ and $\mathcal{MC}$ loci contract, leading to an unambiguous reduction in offense severity at equilibrium; see the left panel of Figure 8. However, the effect on the equilibrium enforcement level is non-monotone. As seen in the right panel of Figure 8, numerical simulations indicate that enforcement effort responds non-monotonically to increases in punishment severity, suggesting that harsher penalties could elicit higher enforcement efforts, highlighting a complementary effect between enforcement effort and punishment. This could help explain recent empirical findings.[25]

[25] Soliman (2022) provides empirical evidence that enforcement effort and punishment may operate as complements. Analyzing a drug policy reform in Kentucky that reduced the size of drug-free school zones—and thus lowered punishment severity—he finds that drug arrests fell by 13% in areas affected by the reform.

8 Concluding Remarks

Motivated by the decentralized nature of law enforcement and the performance-driven discretion exercised by law enforcers, this paper develops a model that captures the interplay between enforcement intensity and crime severity. While much of the existing literature treats these decisions separately, this paper emphasizes that in many—if not all—cases, they are jointly determined. For example, the probability of receiving a speeding ticket is much higher when a driver significantly exceeds the speed limit than when only slightly over it. This pattern reflects a broader and more fundamental insight: the likelihood of detection is shaped not only by the level of enforcement but also by the severity of the offense. Recognizing this feedback mechanism is crucial, as it generates rich interactions between law enforcers and criminals, with significant policy implications. Since law enforcers seek arrests, and arrests depend on the level of crime, policies aimed at directly reducing crime—such as harsher penalties—can backfire: by lowering crime levels, they may inadvertently reduce enforcement intensity, potentially leading to an overall increase in criminal activity.

This paper identifies simple conditions on the detection probability that determine when policies such as increasing penalties or enhancing detection technologies lead to desirable crime-reducing outcomes. These results not only help reconcile empirical findings on crime but also generate new testable predictions. Additionally, the analysis suggests that using arrests as a proxy for law enforcement intensity may not fully capture feedback effects. The relationship is more nuanced than often assumed: it is shown that stronger enforcement incentives may lead to greater effort and yet fewer arrests. As a result, interpreting low arrest rates as evidence of weak enforcement should be approached with caution.

Beyond policy design, the paper examines the unintended consequences of using performance-based metrics—such as arrests—to compensate law enforcers. Under decentralized enforcement, these incentives can still be relatively effective, as agents have no impact on aggregate outcomes. However, when enforcement is centralized, tying compensation to arrests can backfire as enforcers may strategically reduce overall effort to maximize total arrests.

Finally, this study takes the incentives of potential victims as given, embedding them within criminals' expected rewards. A promising direction for future research is to extend the model to incorporate endogenous victim responses, such as increased vigilance or private security. Additionally, the model abstracts away from potential congestion effects in detecting criminals, which is another avenue for further exploration.

A Omitted Proofs

A.1 Proof of Lemma 1

Lemma 1 *For each $\sigma > 0$, there exists a unique enforcement level $e > 0$ with $\mathcal{E}(\mathcal{K}^S(e), \sigma) = e$. This enforcement level is maximal, i.e., $e = 1$, whenever $\mathcal{K}^S(1) \geq \mathcal{K}^D(1; \sigma)$. Moreover, if there is $\bar{\sigma} \in [0, 1]$ solving $\mathcal{K}^S(1) = \mathcal{K}^D(1; \bar{\sigma})$, then $\bar{\sigma}$ is unique and $\mathcal{E}(\mathcal{K}^S(1), \sigma) = 1$ for all $\sigma \geq \bar{\sigma}$; otherwise, $\mathcal{E}(\mathcal{K}^S(e), \sigma) = e \in (0, 1)$ for all $\sigma > 0$.*

Proof: The lemma is proved in three steps.

STEP 1: THE SUPPLY OF CRIME. First, $\mathcal{K}^S(\cdot)$ in (7) is continuous, for $\bar{\omega}(\cdot)$ is continuous, by the Maximum Theorem, and $G(\cdot)$ is atomless. It is also strictly increasing, since $\mathcal{K}^S_e(e) = -g(\bar{\omega})\wp_e(e, \Sigma(e)) < 0$, by the Envelope Theorem. Finally, $\mathcal{K}^S(e) \in (0, 1)$ for all $e \in [0, 1]$.

STEP 2: THE DEMAND FOR CRIME. Define $\bar{\kappa} := c'(1)/[\wp_e(1, \sigma)B]$. As argued in the main text, $e = \mathcal{E}(\kappa; \sigma)$ solves (8) if $\kappa \leq \bar{\kappa}$; otherwise, $\mathcal{E}(\kappa; \sigma) \equiv 1$. Moreover, given σ, $\mathcal{E}(\cdot; \sigma)$ is continuous by the Implicit Function Theorem. It is also strictly increasing in $\kappa \leq \bar{\kappa}$, and thus invertible with inverse $\mathcal{K}^D(\cdot; \sigma) : [0, 1] \to [0, \bar{\kappa}]$; see (9). Finally, notice that $\mathcal{K}^D(0) = 0$.

STEP 3: SINGLE CROSSING. First, if $\mathcal{K}^S(1) \geq \mathcal{K}^D(1; \sigma) = \bar{\kappa}$ then, by Step 2, $\mathcal{E}(\mathcal{K}^S(1); \sigma) = 1$, and hence $e = 1$ solves $\mathcal{E}(\mathcal{K}^S(e), \sigma) = e$. This solution is unique since, for any $e < 1$, $\mathcal{K}^S(e) > \mathcal{K}^S(1) \geq \bar{\kappa}$ and so $\mathcal{E}(\mathcal{K}^S(e); \sigma) = 1$. Moreover, if there is $\bar{\sigma} \in [0, 1]$ solving $\mathcal{K}^S(1) = \mathcal{K}^D(1; \bar{\sigma})$, then $\bar{\sigma}$ is unique because $\mathcal{K}^D(1; \sigma)$ is strictly decreasing in σ; thus, for any $\sigma > \bar{\sigma}$, $\mathcal{K}^S(1) > \mathcal{K}^D(1; \sigma) = \bar{\kappa}$ and hence $\mathcal{E}(\mathcal{K}^S(1); \sigma) = 1$.

Finally, suppose $\mathcal{K}^S(1) < \mathcal{K}^D(1; \sigma) = \bar{\kappa}$. Then, one can apply the Intermediate Value Theorem to $\mathcal{K}^S(\cdot)$ and $\mathcal{K}^D(\cdot; \sigma)$ on $[0, 1]$ to secure existence and uniqueness of $e \in (0, 1)$ such that $\mathcal{K}^S(e) = \mathcal{K}^D(e; \sigma)$, as $\mathcal{K}^S(0) > \mathcal{K}^D(0; \sigma) = 0$ and $\mathcal{K}^D(1; \sigma) > \mathcal{K}^S(1)$. In other words, $\mathcal{E}(\mathcal{K}^S(e), \sigma) = e \in (0, 1)$. This concludes the proof. □

A.2 Proof of Proposition 1

It suffices to show the existence and uniqueness of a pair (σ^*, e^*) such that σ^* is optimal given e^*, and e^* clears the market given σ^*. For then, crime rate and arrest rate are derived outcomes: $\kappa^* = \mathcal{K}^S(e^*)$ and $\alpha^* = \wp(e^*, \sigma^*)\kappa^*$. I next separate into cases depending on the existence of cutoffs $\bar{\sigma}$ and $\underline{e}$, which are defined in Section §4.

CASE 1: $\bar{\sigma}$ EXISTS. Recall that $\bar{\sigma}$ is the unique σ value that solves $\mathcal{K}^S(1) = \mathcal{K}^D(1; \sigma)$.

CASE 1.1: $\bar{\sigma} \leq \Sigma(1)$. In this case, $\mathcal{K}^S(1) \geq \mathcal{K}^D(1; \Sigma(1))$, and thus, by Lemma 1, enforcement $e^* = 1$ clears the market, given $\kappa^* = \mathcal{K}^S(1)$ and severity $\sigma^* = \Sigma(1)$, while severity and criminal entry are optimal given $e^* = 1$. This equilibrium is unique: if there were another one with, say, $e' < 1$, then $\bar{\sigma} \leq \Sigma(e')$ (as Σ is decreasing) and so, by Lemma 1, the market clearing enforcement must be maximal $e' = 1$ given $\sigma = \Sigma(e')$, which is a contradiction.

Case 1.2: $\bar{\sigma} > \Sigma(1)$. For this case, one can use the Intermediate Value Theorem (IVT). Indeed, the market clearing locus is characterized by the increasing and continuous curve $e = \mathcal{E}(\mathcal{K}^S(e); \sigma)$, while the severity locus is simply the decreasing and continuous curve $\sigma = \Sigma(e)$. To apply IVT, I consider the inverse of Σ, denoted by $e_\Sigma : [\Sigma(1), 1] \to [\underline{e}, 1]$, which is well-defined as Σ is strictly decreasing on $[\underline{e}, 1]$. Moreover, when $\sigma = 1$, $e_\Sigma(1) = \underline{e}$ while $e = \mathcal{E}(\mathcal{K}^S(e); 1) = 1 > \underline{e}$; on the other hand, when $\sigma = \Sigma(1)$, $e_\Sigma(\Sigma(1)) = 1$ while $e = \mathcal{E}(\mathcal{K}^S(e); \Sigma(1)) < 1$ by Lemma 1, since $\Sigma(1) < \bar{\sigma}$. Thus, by IVT, there exists a unique $e^* \in (\underline{e}, 1)$ such that $e^* = \mathcal{E}(\mathcal{K}^S(e^*); \Sigma(e^*))$. Note that an equilibrium with $e \in [0, \underline{e}]$ cannot exists, as $\Sigma(e) \equiv 1$ on that domain, while $e = \mathcal{E}(\mathcal{K}^S(e); 1) = 1 > \underline{e}$.

Case 2: $\bar{\sigma}$ does not exist. In this case, $\mathcal{E}(\mathcal{K}^S(e); \sigma) = e \in (0, 1)$ for all $\sigma > 0$. If $\mathcal{E}(\mathcal{K}^S(e); 1) = e \leq \underline{e}$ then $\sigma^* = 1$ and $e^* = \mathcal{E}(\mathcal{K}^S(e^*); 1)$ is an equilibrium. This outcome is unique: if there is another one with $e' > \underline{e}$ then $e' = \mathcal{E}(\mathcal{K}^S(e'); \Sigma(e')) \leq \mathcal{E}(\mathcal{K}^S(e'); 1) \leq \underline{e}$, which is a contradiction. Now, if $\mathcal{E}(\mathcal{K}^S(e); 1) = e > \underline{e}$ then, as previously done, one can apply IVT to functions $e_\Sigma(\sigma) = e$ and $\mathcal{E}(\mathcal{K}^S(e); \sigma) = e$ on the box $[\Sigma(1), 1] \times [\underline{e}, 1]$. □

A.3 Proof of Proposition 2

First, twice differentiating (1) yields $\Pi_{\sigma f} = -\wp_\sigma(e, \sigma) < 0$. Thus, Π is submodular in (σ, f), and so By Topkis (1998), the optimal severity locus $\mathcal{OS}$ shifts left in (σ, e)-space, as seen in the middle panel of Figure 4.

Second, I turn to (κ, e)-space. By the Envelope Theorem in (6), $\bar{\omega}_f = -\wp(\Sigma(e; f), \sigma) < 0$. Hence, the supply of crime $\mathcal{K}^S(e; f)$ shifts left. Since the demand for crime $\mathcal{K}^D$ is unaffected by punishment f, the market clearing enforcement falls along the demand curve (see the left panel of Figure 4). Thus, the market clearing locus $\mathcal{MC}$ shifts down. Altogether, the equilibrium enforcement level e^* unambiguously falls.

Next, observe that the equilibrium severity σ^* falls if the $\mathcal{MC}$ locus shifts down less than $\mathcal{OS}$ does. Fix σ and log-differentiate in f the market clearing $\mathcal{MC}$ locus characterized by $\mathcal{K}^D(e, \sigma) \equiv \mathcal{K}^S(e; f)$, and the severity locus $\mathcal{OS}$ described by $r'(\sigma) - \wp_\sigma(e, \sigma) f \equiv 0$ to get:

$$\left.\frac{de}{df}\right|_{\mathcal{C}^*} \left(\frac{c''}{c'} - \frac{\wp_{ee}}{\wp_e} - \frac{g}{G}\bar{\omega}_e \right) = \frac{g}{G}\bar{\omega}_f \quad \text{and} \quad \left.\frac{de}{df}\right|_{\Sigma^*} = -\frac{\wp_\sigma}{\wp_{\sigma e} f} \tag{13}$$

Since $c'' > 0 > \wp_{ee}$, the slope $de/df|_{\mathcal{MC}} > -(g/G)\bar{\omega}_f/[(g/G)\bar{\omega}_e] = -\bar{\omega}_f/\bar{\omega}_e = -\wp/(\wp_e f)$. On the other hand, $de/d\varphi|_{\mathcal{OS}} \leq -\wp/(\wp_e f)$ if and only if $\wp(e,\sigma)$ is log-submodular:

$$\left.\frac{de}{df}\right|_{\Sigma^*} \leq -\frac{\wp}{\wp_e f} \iff \frac{\wp_{\sigma e}}{\wp_e} - \frac{\wp_\sigma}{\wp} \leq 0. \tag{14}$$

Now, I explore changes in the arrest rate. Using (9), let me define

$$\mathcal{A}(e,\sigma) := \wp(e,\sigma)\mathcal{K}^D(e,\sigma) = c'(e)\wp(e,\sigma)/[\wp_e(e,\sigma)B]$$

Note that $\mathcal{A}$ rises in e (since $\wp_{ee} \leq 0 < c''$ and $\wp_e > 0$) and in σ (since $\wp_e/\wp$ falls in σ as $\wp$ is log-submodular in (e,σ)). Thus, the equilibrium arrest rate $\alpha^* = \mathcal{A}(e^*,\sigma^*)$ falls as f rises.

Finally, I examine the impact on the equilibrium crime rate κ^*. Since an increase in f leads to less policing e^* the net effect on κ^* is unclear. I next show that if $\wp$ is log-modular, then the crime rate must fall. Let me define $\dot{e} := de/df$ and $\dot{\sigma} := d\sigma/df$ as the total derivatives of the equilibrium variables (e,σ) with respect to f. Totally log-differentiating (4) with respect to f yields:

$$\frac{r''}{r'}\dot{\sigma} = \frac{\wp_{\sigma e}}{\wp_\sigma}\dot{e} + \frac{\wp_{\sigma\sigma}}{\wp_\sigma}\dot{\sigma} + \frac{1}{f} \implies \dot{\sigma}\left(\frac{r''}{r'} - \frac{\wp_{\sigma\sigma}}{\wp_\sigma}\right) = \frac{\wp_{\sigma e}}{\wp_\sigma}\dot{e} + \frac{1}{f}.$$

Since $\wp$ is log-modular (and thus log-submodular), it follows that $\dot{\sigma} < 0$. Also, $r'' < 0 \leq \wp_{\sigma\sigma}$, and so $\dot{e} > -\wp_\sigma/(\wp_{\sigma e} f)$. But since $\wp$ is log-modular, $\wp_{\sigma e}/\wp_\sigma = \wp_e/\wp$. Hence,

$$\dot{e} > -\frac{\wp}{\wp_e f} = -\frac{\bar{\omega}_f}{\bar{\omega}_e} \iff \frac{d\bar{\omega}}{df} = \bar{\omega}_e\dot{e} + \bar{\omega}_f < 0$$

Thus, the crime rate $\kappa^* = G(\bar{\omega})$ must fall as f rises. This concludes the proof. □

A.4 Proof of Proposition 4

First, I argue that the optimal severity locus $\mathcal{OS}$ shifts left. Indeed, fix e and notice that $\Sigma(e,\varphi) = \arg\max_\sigma \Pi(\sigma, e|\varphi)$ falls in φ, since $\Pi_{\sigma\varphi} = -(\tilde{\wp}_{\sigma\varphi} f + \tilde{\wp}_\varphi f) = -(\wp_\sigma f + \wp f) < 0$, namely, $\Pi(\cdot)$ in (1) is submodular in (σ,φ). Hence, $\mathcal{OS}$ must shift left in (σ,e)-space.

Next, I examine the market clearing locus $\mathcal{MC}$. First, the demand for crime $\mathcal{K}^D(e,\sigma|\varphi) = c'(e)/[\tilde{\wp}_e(e,\sigma|\varphi)B]$ decreases in φ, as φ rises the marginal efficacy of e (i.e., $\tilde{\wp}_{e\varphi} > 0$). Second, the supply of crime $\mathcal{K}^S(e|\varphi) = G(\bar{\omega}(e|\varphi))$ also decreases in φ, since $\mathcal{K}^S_\varphi = g\bar{\omega}_\varphi = -g\tilde{\wp}_\varphi f = -g\wp f < 0$. As both demand and supply contract, the effect on the market clearing enforcement $e_{\mathcal{C}}(\sigma|\varphi)$ (10) is ambiguous. Clearly, $e_{\mathcal{C}}(\sigma|\varphi)$ falls in φ if the demand shifts left less than the supply does, which is the case provided the supply is sufficiently elastic. To see this, fix $e > 0$ and differentiate $\mathcal{K}^S(e|\varphi)$ and $\mathcal{K}^D(e,\sigma|\varphi)$ in φ to get:

$$\frac{1}{\mathcal{K}^S}\frac{\partial \mathcal{K}^S}{\partial \varphi} - \frac{1}{\mathcal{K}^D}\frac{\partial \mathcal{K}^D}{\partial \varphi} = -\frac{g}{G}\tilde{\wp}_\varphi f + \frac{\tilde{\wp}_{e\varphi}}{\tilde{\wp}_e} = -\frac{g}{G}\wp f + \frac{1}{\varphi} \tag{15}$$

Notice that $\partial e_{\mathcal{C}}/\partial\varphi \leq 0$ if and only if the right side of (15) is negative. In other words, since $\tilde{\wp} \equiv \varphi\wp$, it follows that $\partial e_{\mathcal{C}}/\partial\varphi \leq 0 \iff g(\bar{\omega})/G(\bar{\omega})\tilde{\wp} f \geq 1$. Thus, if the supply curve is sufficiently elastic, the market clearing locus $\mathcal{MC}$ cannot shift up in (σ, e)-space.

As $\mathcal{OS}$ contracts while $\mathcal{MC}$ expands, the equilibrium enforcement level e^* unambiguously falls. However, the effect on the equilibrium severity σ^* is more subtle. I next show that the $\mathcal{OS}$ locus shifts down more than $\mathcal{MC}$ does, leading to a decrease in the offense severity σ^*. To this end, fix σ and log-differentiate $\mathcal{K}^S(e|\varphi) \equiv \mathcal{K}^D(e,\sigma|\varphi)$ and $\Sigma(e,\varphi) \equiv \sigma$ in φ to get:

$$\left.\frac{de}{d\varphi}\right|_{\mathcal{MC}}\left(\frac{c''}{c'} - \frac{\tilde{\wp}_{ee}}{\tilde{\wp}_e} - \frac{g}{G}\bar{\omega}_e\right) = \frac{g}{G}\bar{\omega}_\varphi + \frac{\tilde{\wp}_{e\varphi}}{\tilde{\wp}_e} \quad \text{and} \quad \left.\frac{de}{d\varphi}\right|_{\mathcal{OS}}\left(\frac{\tilde{\wp}_{\sigma e}}{\tilde{\wp}_e}\right) = -\frac{\tilde{\wp}_\varphi}{\tilde{\wp}_e}\left(\frac{\tilde{\wp}_{\sigma\varphi}}{\tilde{\wp}_\varphi}\right)$$

Using that $\tilde{\wp}(e,\sigma|\varphi) \equiv \varphi\wp(e,\sigma)$, the above expression reduces to:

$$\left.\frac{de}{d\varphi}\right|_{\mathcal{MC}}\left(\frac{c''}{c'} - \frac{\wp_{ee}}{\wp_e} + \frac{g}{G}\varphi\wp_e f\right) = -\frac{g}{G}\wp f + 1 \quad \text{and} \quad \left.\frac{de}{d\varphi}\right|_{\mathcal{OS}}\left(\frac{\wp_{\sigma e}}{\wp_e}\right) = -\frac{\wp_\sigma}{\varphi\wp_e} \tag{16}$$

By assumption, $(g/G)\wp f \geq 1$. Also, $c''/c' - \wp_{ee}/\wp_e > 0$, and thus

$$de/d\varphi|_{\mathcal{MC}} \geq -(g/G)\wp f/[(g/G)\varphi\wp_e f] = -\wp/(\varphi\wp_e)$$

On the other hand, $\wp_{\sigma e}/\wp_e \leq \wp_\sigma/\wp$, since $\wp$ is log-submodular in (σ, e). Thus, $de/d\varphi|_{\Sigma^*} \leq -\wp/(\varphi\wp_e)$ by (16). Altogether, $de/d\varphi|_{\mathcal{MC}} \geq de/d\varphi|_{\mathcal{OS}}$, and so severity σ^* falls. □

Now the equilibrium arrest rate. Define

$$\mathcal{A}(e,\sigma,\varphi) := \tilde{\wp}(e,\sigma|\varphi)\mathcal{K}^D(e,\sigma|\varphi) = \frac{c'(e)\tilde{\wp}(e,\sigma|\varphi)}{\tilde{\wp}_e(e,\sigma|\varphi)B}$$

Since $\tilde{\wp} = \varphi\wp$, it follows that $\mathcal{A}(e,\sigma,\varphi) = c'(e)\wp(e,\sigma)/[\wp_e(e,\sigma)B]$. Thus, $\alpha(e,\sigma,\varphi)$ depends on e,σ only, and because equilibrium e^* and σ^* fall as φ rises, it follows that the equilibrium arrest rate must also fall, since $\wp_e/\wp$ falls in e and in σ, as $\wp$ is increasing and concave in e, and log-submodular in (e,σ). □

Finally, the equilibrium crime rate. For this comparative static, it is further assumed that $\wp$ is log-modular in (e,σ). This is sufficient to ensure that the effect of technology on criminal entry is not crowded out by a reduction in the level of enforcement.

Claim A.1 *Suppose that $\wp$ is log-modular in (e, σ). If the detection technology improves (i.e., φ rises), then the equilibrium crime rate κ^* falls.*

Proof: Let $\Delta\mathcal{K}(e, \sigma|\varphi) := \mathcal{K}^S(e|\varphi) - \mathcal{K}^D(e, \sigma|\varphi)$ denote the *excess of supply*. Notice that $\mathcal{MC}$ locus is determined by $\Delta\mathcal{K} \equiv 0$. Totally differentiating $\Delta\mathcal{K}(e, \sigma|\varphi) \equiv 0$ and $\Sigma(e|\varphi) \equiv \sigma$ leads to the following 2×2 system:

$$\begin{pmatrix} \Pi_{\sigma e} & \Pi_{\sigma\sigma} \\ \Delta\mathcal{K}_e & \Delta\mathcal{K}_\sigma \end{pmatrix} \begin{pmatrix} e_\varphi \\ \sigma_\varphi \end{pmatrix} = \begin{pmatrix} -\Pi_{\sigma\varphi} \\ -\Delta\mathcal{K}_\varphi \end{pmatrix} \implies \begin{pmatrix} e_\varphi \\ \sigma_\varphi \end{pmatrix} = det^{-1} \begin{pmatrix} \Delta\mathcal{K}_\sigma & -\Pi_{\sigma\sigma} \\ -\Delta\mathcal{K}_e & \Pi_{\sigma e} \end{pmatrix} \begin{pmatrix} -\Pi_{\sigma\varphi} \\ -\Delta\mathcal{K}_\varphi \end{pmatrix} \tag{17}$$

where $det \equiv \Pi_{\sigma e}\Delta\mathcal{K}_\sigma - \Pi_{\sigma\sigma}\Delta\mathcal{K}_e < 0$, since $\Delta\mathcal{K}_\sigma = -\mathcal{K}^D_\sigma > 0$, by (9), and $\Delta\mathcal{K}_e = \mathcal{K}^S_e - \mathcal{K}^D_e < 0$, by (7) and (9). Next, notice that, in equilibrium, the crime rate falls if $d\mathcal{K}^S/d\varphi = g(\bar{\omega}_e e_\varphi + \bar{\omega}_\varphi) < 0$. Since density g is positive, one needs to examine the sign of $\bar{\omega}_e e_\varphi + \bar{\omega}_\varphi$. Using the expression for e_φ from (17), and doing some straightforward algebra yields:

$$\bar{\omega}_e e_\varphi + \bar{\omega}_\varphi < 0 \iff \underbrace{\Delta\mathcal{K}_\sigma(\Pi_{\sigma e}\bar{\omega}_\varphi - \Pi_{\sigma\varphi}\bar{\omega}_e)}_{(\clubsuit)} + \underbrace{\Pi_{\sigma\sigma}(\Delta\mathcal{K}_\varphi\bar{\omega}_e - \Delta\mathcal{K}_e\bar{\omega}_\varphi)}_{(\spadesuit)} > 0$$

Now, observe that $\mathcal{K}^S_\varphi\bar{\omega}_e \equiv \mathcal{K}^S_e\bar{\omega}_\varphi$, given (6) and (7). Thus, $\Delta\mathcal{K}_\varphi\bar{\omega}_e - \Delta\mathcal{K}_e\bar{\omega}_\varphi = \mathcal{K}^D_e\bar{\omega}_\varphi - \mathcal{K}^D_\varphi\bar{\omega}_e < 0$, since $\mathcal{K}^D_e > 0 > \mathcal{K}^D_\varphi, \bar{\omega}_\varphi, \bar{\omega}_e$. Consequently, $(\spadesuit) > 0$ since $\Pi_{\sigma\sigma} < 0$.

Hence, to show that the crime rate falls, it is enough to show that $(\clubsuit) \geq 0$. But the latter reduces to showing that $\Pi_{\sigma e}\bar{\omega}_\varphi - \Pi_{\sigma\varphi}\bar{\omega}_e \geq 0$, since $\Delta\mathcal{K}_\sigma > 0$. Now, recall that $\Pi_{\sigma e} = -\varphi\wp_{\sigma e}f$; $\Pi_{\sigma\varphi} = -\wp_\sigma f$; $\bar{\omega}_\varphi = -\wp f$; and $\bar{\omega}_e = -\varphi\wp_e f$. Therefore,

$$\Pi_{\sigma e}\bar{\omega}_\varphi - \Pi_{\sigma\varphi}\bar{\omega}_e = \varphi f^2(\wp_{\sigma e}\wp - \wp_e\wp_\sigma) = 0,$$

where last equality holds, since $\wp$ is log-modular in (σ, e). This concludes the proof. □

A.5 Proof of Proposition 5

In this section, the proof of Proposition 5 is finished. As argued in the main text, it remains to show that the detection probability $\wp(e^*, \sigma^*)$ falls with B, where $\sigma^* = \Sigma(e^*)$.

To this end, let $\Sigma(e) < 1$ and write the first-order condition (4) as $r'(\Sigma(e)) \equiv \wp_\sigma(e, \Sigma(e))f$. Log-differentiate both sides in e to obtain:

$$\frac{r''}{r'} \times \Sigma' = \frac{\wp_{\sigma e}}{\wp_\sigma} + \frac{\wp_{\sigma\sigma}}{\wp_\sigma} \times \Sigma' \implies \Sigma' = -\frac{\wp_{\sigma e}}{\wp_\sigma} \times \left(\frac{\wp_{\sigma\sigma}}{\wp_\sigma} - \frac{r''}{r'}\right)^{-1}$$

Next, differentiate $\wp(e, \Sigma(e))$ to get:

$$\frac{d\wp}{de} = \wp_e + \wp_\sigma \times \Sigma' = \wp_e - \wp_{\sigma e} \times \left(\frac{\wp_{\sigma\sigma}}{\wp_\sigma} - \frac{r''}{r'}\right)^{-1}$$

Therefore, it is immediate to see that $d\wp/de \leq 0$ if and only if (12) holds.

To finalize the proof, I show that if $\wp$ is log-modular in (e, σ), and $\wp$ and r are isoelastic in σ, then (12) holds. First, if $\wp$ is log-modular, then $\wp$ must be of the form $\wp(e, \sigma) = y(e) \times z(\sigma)$. Thus, $\wp_{\sigma e}/\wp_e = \wp_\sigma/\wp$. In addition, if $\wp$ is isoelastic in σ, then z is isoelastic in σ and thus must be proportional to σ^k, with $k \geq 1$, as $\wp$ must be convex in σ. Similarly, r must be proportional to σ^q, with $q \in (0, 1)$, as r must be strictly concave in σ. Altogether,

$$\frac{\wp_{\sigma\sigma}}{\wp_\sigma} - \frac{\wp_{\sigma e}}{\wp_e} - \frac{r''}{r'} = \frac{\wp_{\sigma\sigma}}{\wp_\sigma} - \frac{\wp_\sigma}{\wp} - \frac{r''}{r'} = -\frac{1}{\sigma} - \frac{(q-1)}{\sigma} = -\frac{q}{\sigma} < 0.$$

This concludes the proof. □

B Convex Penalties

I now explore the case where penalties are allowed to vary with the severity of an offense. To this end, smoothly index the fine $f(\sigma|\varphi)$ so that $f_\varphi(\sigma|\varphi) > 0$, where $\varphi \in \mathbb{R}$. When the fine $f(\sigma|\varphi)$ is log-supermodular in (σ, φ), a rise in φ raises the elasticity of the fine in the offense severity σ.[26] I refer to a greater φ as *harsher marginal punishments.*

Proposition B.1 (Convex Punishment) *If penalties are marginally harsher, then the enforcement level e^* falls. The offense severity σ^* and the arrest rate α^* fall, both fall, provided the detection chance $\wp(e, \sigma)$ is log-submodular in (e, σ). The crime rate κ^* falls if the detection chance $\wp(e, \sigma)$ is modular in $\wp(e, \sigma)$.*

Proof: First, since $f_\varphi > 0$ and f log-supermodular in (σ, φ), the fine f is supermodular in (σ, φ), or $f'_\varphi > 0$, by Topkis (1998). Next, twice differentiating (1) yields $\Pi_{\varphi\sigma} = -\wp_\sigma(e, \sigma) f_\varphi(\sigma|\varphi) - \wp(e, \sigma) f'_\varphi(\sigma|\varphi)$. Thus, Π is submodular in (σ, φ), since $f'_\varphi > 0$. So By Topkis (1998), the optimal severity locus $\Sigma^*(e|\varphi) \in \arg\max_\sigma \Pi(\sigma, e|\varphi)$ shifts left.

Next, I turn to (κ, e)-space. By the Envelope Theorem in (6), $\bar{\omega}_\varphi = -\wp(e, \sigma) f_\varphi(\sigma|\varphi) < 0$, where $\sigma = \Sigma^*(e)$. Hence, the supply of crime $\mathcal{K}^S(e|\varphi)$ shifts left in φ. Since the demand for

[26] For instance, this property holds if φ raises the convexity of the fine f in the sense of Arrow-Pratt. Indeed, a rise in φ is associated with a more convex fine iff the marginal fine $f'(\sigma|\varphi)$ is log-supermodular (Diamond and Stiglitz, 1974). But then, this implies that the fine $f(\sigma|\varphi)$ is log-supermodular in (σ, φ). For let $\mathbb{I}(\sigma, \sigma') \equiv 1$ if $\sigma' \leq \sigma$ and 0 otherwise. Since $\mathbb{I}(\sigma, \sigma')$ is log-supermodular in (σ, σ'), the function $\mathbb{I}(\cdot) f'(\cdot)$ is log-supermodular in $(\sigma, \sigma', \varphi)$; and thus, $f(\sigma|\varphi) = \int_0^\infty \mathbb{I}(\sigma, \sigma') f'(\sigma'|\varphi) d\sigma'$ is log-supermodular in (σ, φ), since log-supermodularity is preserved under integration.

crime $\mathcal{K}^D$ is constant in φ, the market clearing enforcement falls along the demand curve; thus, the market clearing locus $\mathcal{MC}$ shifts down. Thus, enforcement e^* unambiguously falls.

Next, observe that the equilibrium severity σ falls if the $\mathcal{MC}$ locus shifts down less than $\mathcal{OS}$, which is the case here. Indeed, fix σ and log-differentiate the market clearing $\mathcal{MC}$ locus $\mathcal{K}^D(e,\sigma) \equiv \mathcal{K}^S(e|\varphi)$, and the severity $\mathcal{OS}$ locus $r'(\sigma) \equiv \wp_\sigma(e,\sigma)f(\sigma|\varphi) - \wp(e,\sigma)f'(\sigma|\varphi)$ to get:

$$\left.\frac{de}{d\varphi}\right|_{\mathcal{MC}} \left(\frac{c''}{c'} - \frac{\wp_{ee}}{\wp_e} - \frac{g}{G}\bar{\omega}'\right) = \frac{g}{G}\bar{\omega}_\varphi \quad \text{and} \quad \left.\frac{de}{d\varphi}\right|_{\mathcal{OS}} \left(\frac{\wp_{\sigma e}}{\wp_e} + \frac{f'}{f}\right) = -\frac{\wp f_\varphi}{\wp_e f}\left(\frac{\wp_\sigma}{\wp} + \frac{f'_\varphi}{f_\varphi}\right) \tag{18}$$

Since $c'' > 0 > \wp_{ee}$, the slope $de/d\varphi|_{\mathcal{MC}} > -(g/G)\bar{\omega}_\varphi/[(g/G)\bar{\omega}'] = -\bar{\omega}_\varphi/\bar{\omega}' = -\wp f_\varphi/(\wp_e f)$. Thus, $de/d\varphi|_{\mathcal{MC}} > -\wp f_\varphi/(\wp_e f)$. By looking at $de/d\varphi|_{\mathcal{OS}}$ in (18), a sufficient condition to ensure that the equilibrium severity falls is:

$$\frac{\wp_\sigma}{\wp} + \frac{f'_\varphi}{f_\varphi} \geq \frac{\wp_{\sigma e}}{\wp_e} + \frac{f'}{f} \iff \frac{f'_\varphi}{f_\varphi} - \frac{f'}{f} \geq \frac{\wp_{\sigma e}}{\wp_e} - \frac{\wp_\sigma}{\wp} \tag{19}$$

For then $de/d\varphi|_{\mathcal{OS}} \leq -\wp f_\varphi/(\wp_e f) < de/d\varphi|_{\mathcal{MC}}$. Notice that (19) holds, since $\wp(e,\sigma)$ is log-submodular—and so the right side of the right expression in (19) is negative—and $f(\sigma|\varphi)$ is log-supermodular—and so the left side of the right expression in (19) is positive.

Now, let us explore changes in the arrest rate. Slightly abusing notation, let $\alpha(e,\sigma) \equiv \wp(e,\sigma)\mathcal{K}^D(e,\sigma) = c'(e)\wp(e,\sigma)/[\wp_e(e,\sigma|\varphi)B]$. Note that α rises in e (for $\wp_{ee} < 0 < c''$) and in σ (for $\wp_e/\wp$ falls in σ). Thus, $\alpha(e(\varphi),\sigma(\varphi))$ falls as φ rises, since $e'(\varphi), \sigma'(\varphi) < 0$.

The effects on the crime rate are more difficult to generalize to the case of convex penalties. However, if $\wp$ is modular then $\wp_{e\sigma} = 0$. In such case, the demand curve $\mathcal{K}^D$ in (9) is unaffected by the offense severity, and thus, the demand $\mathcal{K}^D$ falls in φ, since the enforcement level falls in φ. As a result, the crime rate $\kappa = \mathcal{K}^D(e|\sigma)$ falls in φ. □

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