Optimal Policing with (and without) Criminal Search*

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Abstract

We develop a crime and policing model, in which a police agency elects how to allocate scarce resources across neighborhoods to minimize total crime, while potential criminals choose whether, where, and, in some cases, when to commit a crime. Neighborhoods vary by their levels of private protection (vigilance) and potential criminal rewards. We characterize the optimal policing allocation in two scenarios.

In the first scenario, criminals engage in an active search process to identify the most favorable target, while in the second one, they make decisions solely based on their prior knowledge due to technological constraints or limited expertise. With criminal search, the optimal police allocation depends on the difference in vigilance levels across neighborhoods, assigning more resources to low vigilance neighborhoods. Conversely, in the absence of criminal search, the allocation strategy depends on the degree of rent inequality among neighborhoods, with a priority placed on allocating resources to neighborhoods with higher rents. We also identify conditions under which policing all neighborhoods equally is optimal. Our findings underscore that an optimal policing design must not only consider neighborhood characteristics but also other factors that may impact criminals' decision-making, including whether they engage in active search.

Keywords: Crime rate, optimal policing, vigilance inequality, rent inequality, search, displacement, deterrence.

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1 Introduction

Recent estimates indicate that the US spends around 150 billion dollars annually on police protection — an amount that has risen around 21% over the last decade (Anderson, 2021). Lately, as part of its 2023 budget, President Biden's administration requested a fully paid-for new investment of approximately $37 billion to support law enforcement and crime prevention.[1] The landmark paper of Becker (1968) predicts that an increase in police resources should lead to less crime. The logic follows from analyzing the decision-making of potential criminals, who compare the expected utility of committing a crime to their outside option; therefore, policing has a *deterrent* effect since it intuitively makes the outside option relatively more attractive. Understanding the deterrent effect of policing is an important question in the empirical literature; see, e.g., Chalfin and McCrary (2017) for a comprehensive survey.

An important assumption that Becker's model relies on is that the outside option equals the utility of abstaining from crime. Yet, it is possible that a potential offender's outside option reflects the possibility of committing a crime elsewhere. Thus, increasing policing in one area may just lead to more crime somewhere else, resulting in a phenomenon known as *displacement*.[2] Recent estimates indicate that displacement effects can be quite large: Maheshri and Mastrobuoni (2020) find that an extra guard lowers the likelihood a bank is robbed by 35% to 40%, with over half of this reduction being displaced to nearby unguarded banks. Likewise, Blattman et al. (2021) worked with the city of Bogotá (Colombia) to design a large security experiment that randomly assigned double police patrol time to "hot spot" streets, finding that crime mildly decreased on the treated streets but increased on nearby streets, especially for property crime.[3,4] These findings indicate that (i) potential criminals contemplate many crime targets before making a decision; and (ii) lowering the payoff of one potential crime target (e.g., with more policing) may simply make other ones more enticing.

In this paper, we ask: how should scarce policing resources be allocated across *many* potential crime targets if the goal is to reduce crime? To this end, we develop a model of

[1] https://www.whitehouse.gov/briefing-room/statements-releases/2022/08/01/fact-sheet-president-bidens-safer-america-plan-2/.

[2] The criminology literature has also argued that increased policing in a particular area could lead to less crime in nearby areas through diffusion of benefits, e.g., an increase in deterrence beyond the targeted area (Weisburd and Telep, 2014). This channel, while certainly plausible, is not supported empirically by the most recent economics literature (e.g., Maheshri and Mastrobuoni, 2020; Blattman et al., 2021).

[3] Property crime is intuitively the one most responsive to incentives. Also, it constitutes around two-thirds of total crime in the US and Europe (Buonanno et al., 2011).

[4] Aside from spatial displacement, Vollaard (2017) provides empirical evidence that marine oil pollution crimes are temporally displaced to night times when the probability of conviction is lower due to technology constraints. Yang (2008) examines tactical displacement in the context of customs reform in the Philippines, finding evidence that increasing enforcement in customs against a particular method of avoiding import duties lowers the targeted method but substantially raises an alternative duty-avoidance method.

policing and crime that accounts for both deterrence and displacement. We consider a unit mass of *potential criminals*, and a finite but arbitrary number of "*neighborhoods*" to represent that potential criminals can choose not only *whether* to commit a crime, but also *where* and (possibly) *when* to do so. We assume neighborhoods differ across several dimensions. First, they vary by their private *vigilance* levels.[5] Specifically, each neighborhood may or may not be privately protected (high vs. low vigilance). If not protected, then an attempted crime succeeds (e.g., a criminal successfully breaks into a house and steals the victim's property); otherwise, it fails. The prior failure chance of an attempted crime is specific to the neighborhood. Second, neighborhoods differ by their potential criminal *rewards*, namely, the amount that the criminal can steal if successful.[6] Finally, on top of vigilance, a *police agency* allocates a fixed amount of policing resources across these neighborhoods to minimize total crime, namely, the sum of crime rates across neighborhoods. We model policing as a crime deterrent:[7] from the criminals' perspective, attempting a crime triggers the event of being apprehended, and thereby punished. The chance of this latter event, i.e., the *capture probability*, increases with the policing resources allocated to the neighborhood.

We examine subgame-perfect equilibria, in which the police agency allocates policing resources to minimize total crime, and then potential criminals choose whether and, if so, where to attempt a crime. We study two polar cases, aiming to reflect the potential criminals' "modus operandi." In our leading *criminal-search case*, we consider criminals who can costly acquire information about neighborhoods before deciding whether to attempt a crime or not. This sequential decision process leads to an additional choice of not only whether and where to attempt a crime but also when to do so.[8] In the *no-criminal-search case*, the latter decision is absent, reflecting a prohibitively costly information acquisition process, which may owe to lack of criminal expertise, or to security measures being hard-to-detect by criminals. The analysis of these two cases lead to qualitatively different results on how police resources should be allocated. From a technical perspective, the two cases are not isomorphic to

[5] Anderson (2021) estimates that the US spends around \$113 billion annually on private security measures such as security systems (\$54.8 billion), safety lightning (\$14 billion), locks, safes, vaults, and locksmiths (\$10.5 billion), protective fences (\$3.5 billion), and guards and patrol (\$30.6 billion).

[6] Using detailed data on stolen property goods and prices in the UK, Draca et al. (2019) provides evidence that property crime is responsive to the rewards of crime, adjusting relatively fast to changes in these ones. In the US, transfers from victims to criminals are of the order of \$3.6 billion for household theft, \$2.4 billion for household burglary, and \$1.1 billion for motor vehicle theft (Anderson, 2021).

[7] In practice, the act of policing involves responding to calls for service, engaging in patrol, and investigating crimes (Owens, 2020). Using detailed geographic data on patrol cars in Dallas, Weisburd (2021) documents that police presence, or patrolling, has a significant impact on criminal behavior. Policing may also involve direct interactions with citizens, perhaps at a social cost (e.g., the stop-and-frisk program in New York); see Manski and Nagin (2017) for a policing model that focuses on confrontational tactics.

[8] The criminology literature suggests that most burglars are experts and follow a sequential decision-making process (Nee and Meenaghan, 2006).

each other and call for different solution approaches, which highlights the different nature of trade-offs faced by the police agency depending on whether criminals engage in search.

More precisely, our leading case takes the form of a sequential search problem for the criminal. Specifically, upon paying an inspection cost, a potential criminal can observe both whether a neighborhood is privately protected and whether it is policed. After learning the neighborhood state, the criminal can choose to: (i) commit a crime; (ii) move on to another neighborhood; or (iii) return to a previously explored one. As a result, a potential criminal's strategy consists of an action plan that specifies where to start the search (selection rule) and when to end it (stopping rule).[9] We leverage results from the sequential search literature (Weitzman, 1979); in particular, we solve the criminals' decision problem in two steps. First, we apply Weitzman's optimal search rule, which consists of sorting neighborhoods from top to bottom according to some "indexes," and then proceeding sequentially until the realized payoff in a neighborhood exceeds both the indexes in all not inspected neighborhoods and the highest realized payoff seen so far.[10] Second, we use recent advances in the consumer search literature (e.g., Choi et al., 2018) to recast the criminal search problem as a discrete choice problem, allowing us to tractably compute the crime rate in each neighborhood as a function of the policing allocation and the optimal criminal search strategy.[11]

To characterize the criminals' optimal search behavior, we focus on settings in which criminals' reservation utility from non-crime related activities is not too low so that criminals can potentially be deterred from crime. Likewise, to ensure that all neighborhoods are attractive to the criminal before the search takes place, we posit that the inspection cost relative to the criminal reward is low enough in all neighborhoods. That said, we find that criminals rank a neighborhood higher if it has a higher chance of being unprotected and unpoliced, and if it offers higher rewards. Thus, all else equal, increasing policing in one neighborhood would naturally displace crime to other neighborhoods by lowering its ranking. As for stopping, criminals find it optimal to commit a crime as soon as they encounter an unprotected and unpoliced neighborhood (Proposition 1); otherwise, the search continues. If all neighborhoods are policed and protected, then the criminal is effectively *deterred*.

[9] Thus, given a police allocation, the criminals' behavior is effectively predictable. Recently, using detailed micro-level data on commercial robberies against businesses in Milan (Italy), Mastrobuoni (2020) documents that robbers (who are believed to be professional criminals) display predictable behavioral patterns.

[10] In Weitzman (1979), a searcher faces a pool of boxes, each having an uncertain payoff drawn from an exogenous distribution. The searcher can open a box at a cost to learn its payoffs, and then choose whether to stop, recall, or keep searching. From a theoretical viewpoint, a novelty of our work is that information is endogenous: the distribution of payoffs for each box depends on the chosen policing allocation.

[11] In the ordered search literature (e.g., Armstrong, 2017), the non-stationary consumer search problem is transformed into a discrete choice problem to compute market shares. This approach has been proven useful to guide empirical work in industrial organization (e.g., Moraga-González et al., 2023).

We leverage the optimal criminal search rule to transform the problem into a discrete choice one (Proposition 2). We show that, given a policing allocation, the crime rate in neighborhood n equals the chance that all neighborhoods inspected before it were either policed or protected times the chance that neighborhood n is not (Lemma 2). This highlights that the crime rate in a neighborhood not only depends on its own vigilance and policing levels but also on the vigilance and policing levels of all neighborhoods inspected beforehand.

Having determined the crime rates in each neighborhood, we then turn to our main goal: the optimal allocation of scarce policing resources. The problem of the police agency can be seen as minimizing the sum of crime rates, subject to a resource constraint that the sum of resources allocated to each neighborhood cannot exceed an exogenously fixed amount. A key challenge in this optimization problem is that a policing allocation impacts the criminal's optimal selection rule, which, in turn, affects the crime rates in all neighborhoods. To address this obstacle, we consider a probabilistic reformulation of the problem and show that the total crime rate can be seen as the probability that there is a "success" among a fixed number of independent, albeit heterogeneous trials (Proposition 3). We prove that the total crime rate is unaffected by the order by which neighborhoods are inspected. Thus, the police agency problem can be recast as maximizing the total *deterrence rate* — the probability that all neighborhoods are either policed or protected — subject to the aforementioned resource constraint. This yields a well-behaved non-linear programming problem, which can be solved in closed-form using standard tools (Proposition 4).

We find that optimal police allocation depends on the degree of *vigilance inequality* across neighborhoods. Indeed, if (and only if) all neighborhoods are equally protected in the sense of having the same vigilance level, the optimal policing allocation is *fair*: all neighborhoods are policed at the same intensity (Corollary 1). Otherwise, the optimal allocation entails *selection*: only a subset of neighborhoods are policed, namely those with low vigilance, resulting in a compensating effect of public policing for private vigilance. In general, to determine whether neighborhood n should be policed, the police agency must compare a fair division of available resources against the average vigilance gap between n and all neighborhoods with lower vigilance than n. Neighborhood n is policed if and only if the fair division exceeds this gap. As for the policing *intensity* (within policed neighborhoods), the optimal allocation satisfies the familiar "bang-per-buck" principle from consumer theory: the increase in deterrence in a neighborhood owed to a one-unit increase in policing must be the same across all policed neighborhoods. Thus, a compensation scheme follows: the police agency allocates a fair share of resources to each neighborhood minus/plus an amount that equals the excess/deficit in vigilance of the neighborhood compared to the average.

Finally, we fully solve the no-criminal-search case. There, potential criminals just choose whether and where to attempt a crime by comparing the ex-ante profits from crime (expected rewards minus expected penalty) associated with each neighborhood. We show that, in equilibrium, policed neighborhoods are those that offer the highest expected profits to criminals (Lemma 3); moreover, the set of policed neighborhoods must have the property that adding a neighborhood to or removing it from this set would induce criminals to attempt a crime elsewhere (Proposition 5). Contrary to the criminal-search case, the optimal policing equalizes expected criminal profits across policed neighborhoods, rather than the bang-per-buck. Thus, the optimal policing depends critically on the amount of inequality in the expected rewards across neighborhoods, which we refer to as *rent inequality*.

In particular, fair policing is optimal only when all neighborhoods offer the same rents (i.e., expected rewards) to criminals (Corollary 2); otherwise, the optimal policing increasingly allocates resources to neighborhoods with higher rents. This holds even if all neighborhoods share the same vigilance levels. As in the criminal-search case, the optimal policing assigns a fair share of resources to each policed neighborhood minus/plus an amount that is proportional to the excess/deficit in rents of the neighborhood compared to the average (Proposition 6). This means that policed neighborhoods with low/high rents (relative to the average) are allocated less/more than a fair share of resources.

We organize the paper as follows. Section §2 reviews the literature and §3 sets up the model. In §4 we characterize the optimal criminal search. Sections §5 and §6 characterize the optimal policing allocation for the search and no-search case, respectively. We conclude in §7. All omitted proofs and analyses are in the Appendix.

2 Literature Review

Our paper relates to several strands of the literature. First, as we study policies aimed at minimizing crime, our work relates to the normative theoretical literature on the public enforcement of the law; see, e.g., Garoupa (1997) and Polinsky and Shavell (2000) for excellent surveys. This literature considers an enforcement authority that chooses both the capture chance and the punishment size to maximize a social welfare function, and it mostly focuses on deterrence by extending Becker (1968), abstracting from displacement considerations. In contrast, in our paper (i) criminals choose among many crime targets; and (ii) penalties are fixed from the outset. This unveils the type of neighborhood data that is needed to improve the allocation of police resources to decrease crime, and also how the optimal allocation depends on the criminals' operation practices, which, to our knowledge, is new in the literature.

Second, our work contributes to literature that analyzes the interaction between the police and potential criminals. Eeckhout et al. (2010) examine optimal policing strategies that minimize crime and find that random crackdowns, or publicly announcing to police identical groups at different intensities, can be optimal. Persico (2002) examines a model in which two groups of heterogeneous citizens can be policed at different rates, and provides conditions under which fair policing (i.e., policing both groups at the same intensity) minimizes crime. Fu and Wolpin (2018) develop a general model of crime, in which there is an arbitrary number of neighborhoods (cities), a continuum of potential criminals (citizens), and a police agency (government) in each city that acts as a Stackelberg leader. A key distinction is that neighborhoods are modeled as closed economies in their paper, impeding criminals to move across them, which is a feature that lies at the heart of our framework.

A growing literature examines theoretically how crime can move across potential crime targets. Draca et al. (2019) develop a model in which potential criminals choose one of two goods to steal and can switch depending on the capture probabilities associated with each good. In a model with an arbitrary number of heterogeneous potential crime targets, Helsley and Strange (2005) study the displacement effect of vigilance and the substitution between vigilance and policing using a game-theoretic framework. Their model is rich in that both vigilance and policing are endogenous; however, it assumes that all targets are policed at the same intensity, which we show only occurs in very specific circumstances. There is a related literature that examines how individual vigilance choices may displace crime to other potential victims; see, e.g., Vásquez (2022) and references therein.[12]

Finally, our work connects to the literature that incorporates search features into crime models. In labor markets, crime can be seen as an alternative to formal work. Burdett et al. (2003, 2004), Huang et al. (2004), and Engelhardt et al. (2008) incorporate crime into a job search model where agents search for jobs while facing opportunities to commit a crime. İmrohoroğlu et al. (2004) develop a dynamic model in which heterogeneous agents face stochastic employment opportunities and decide whether to engage in criminal activities based on their employment status. With a focus on how criminals' discount factors interplay with deterrence policies, Lee and McCrary (2017) develop a stationary search model a la McCall (1970) with exogenous policing, in which potential criminals face a single criminal opportunity in every period and decide whether to take it or wait for a better draw. The question of optimal policing is absent in this body of work.

[12] Draca and Machin (2015) provides a recent survey of the empirical literature. Recently, Matheson et al. (2023) examine how the rise of working from home (during and after the UK lockdown in 2020) impact burglaries, finding that burglaries relocated to neighborhoods with low rates of working from home.

3 The Model

Consider a unit-mass continuum of potential criminals. There is a finite number of *neighborhoods* $n \in \mathcal{N} = \{1, \dots, N\}$, with $N > 1$, to potentially commit a crime. Each neighborhood n can be either privately protected, policed, or both. The respective prior probabilities that n is protected and policed are $\phi_n \in [0, 1)$ and $p_n \in [0, 1]$. A crime attempted in a privately protected neighborhood is guaranteed to fail, while it leads to a sure arrest in a policed neighborhood. Thus, ϕ_n and p_n can be seen as the *failure chance* of an attempted crime in n and the *probability of being caught* by the police in n, respectively.[13] Whether a neighborhood is policed, protected, or both is unknown to the criminal without inspection. Hence, from an ex-ante perspective, if a criminal attempts a crime in neighborhood n, then he succeeds and is not arrested with chances $1 - \phi_n$ and $1 - p_n$, respectively. If the criminal succeeds, then he obtains a *reward* $r_n > 0$; however, if he is captured by the police, he pays a *fine* $f > r_n$. Lastly, if the criminal fails and is not captured, he gets 0. Criminals are risk-neutral and have an outside option u_0 which is normalized to zero ($u_0 = 0$).[14]

Following the sequential search literature (Weitzman, 1979), after paying an *inspection cost* $c > 0$, the criminal learns whether the neighborhood is policed or protected. He then chooses whether to (i) commit a crime, (ii) move on to another neighborhood, (iii) return to a previously inspected neighborhood,[15] or (iv) take the outside option.[16] The *crime rate in neighborhood* n, denoted by $\kappa_n \in [0, 1]$, is the chance that a crime is committed in n.

The interaction transpires in stages. First, the police agency chooses a policing allocation $(p_n)_{n\in\mathcal{N}}$ to minimize the *total crime rate* $\sum_{n\in\mathcal{N}} \kappa_n$ subject to a *resource constraint* $\sum_{n\in\mathcal{N}} p_n \leq \bar{p}$ with $\bar{p} \in (0, 1)$ denoting the available policing resources. Second, each potential criminal chooses a strategy consisting of *where to start* the search (selection rule) and *when to end* it (stopping rule). We study subgame perfect Nash equilibria of this interaction.

4 Optimal Criminal Search

To solve the criminals' problem, we apply Weitzman's optimal search rule. This requires finding *reservation indexes* z_n for each neighborhood $n \in \mathcal{N}$. Intuitively, these indexes

[13] More specifically, we envision the probability of being caught and failure probability in neighborhood n as random variables with respective averages p_n and ϕ_n. From this viewpoint, the model assumes that these random variables take values 0 and 1. This simplification not only renders the model tractable, but also it let us provide a clean comparison between the search and no search cases, which is one of our main goals.

[14] Our results immediately generalize to settings in which $r_n > u_0 > r_n - f$ for all n.

[15] As standard, the cost of re-visiting a previously inspected neighborhood is zero (costless recall).

[16] See Doval (2018) for a general analysis of the case in which the searcher can also choose to take any uninspected box without costly learning its payoff first. This, in general, makes the search problem intractable.

reflect the payoff that makes the criminal indifferent between searching and not. Specifically, after paying the inspection cost $c > 0$, the criminal learns the crime payoff associated with neighborhood n, and then optimally chooses whether to commit a crime in n or move elsewhere. Thus, the reservation index z_n solves the following recursive equation:

$$\begin{aligned} z_n = -c + \max\{z_n, r_n\}(1-\phi_n)(1-p_n) + \max\{z_n, r_n - f\}(1-\phi_n)p_n \\ + \max\{z_n, 0\}\phi_n(1-p_n) + \max\{z_n, -f\}\phi_n p_n. \end{aligned} \tag{1}$$

To understand (1), suppose the criminal inspects neighborhood n, incurring a cost $c > 0$. Then, the criminal will learn whether the neighborhood is policed or not, and whether it is privately protected or not. The criminal will then make an optimal decision whether to commit a crime or continue exploring. The right hand side of (1) reflects the ex-ante payoff of inspecting neighborhood n and learning the associated payoff of attempting a crime, whereas the left hand side denotes the payoff of moving on to the next alternative. Of course, if inspection cost c is too high, the criminal will never choose to keep searching; also, because criminals have an outside option, each will consider committing a crime in n as long as $z_n \geq 0$. To avoid trivialities, we henceforth restrict the model parameters to satisfy:

Assumption 1. *Parameters $(\phi_n, r_n, c, \bar{p})$ satisfy $c \leq r_n(1-\phi_n)(1-\bar{p})$ for all $n \in \mathcal{N}$.*

The next lemma characterizes the reservation indexes z_n for all neighborhoods n.

Lemma 1. *The reservation index associated with neighborhood n obeys $z_n \in (0, r_n)$ and:*

$$z_n = \frac{-c + r_n(1-\phi_n)(1-p_n)}{(1-\phi_n)(1-p_n)}. \tag{2}$$

Assumption 1 ensures that, for each neighborhood n, the reservation index z_n is strictly positive and less than the reward r_n, i.e., $z_n \in (0, r_n)$. Thus, all neighborhoods are attractive to the criminal from an ex-ante viewpoint. Notice that reservation index z_n in (2) exhibits natural properties: it increases with reward r_n, and decreases with capture probability p_n and failure chance ϕ_n. In other words, criminals rank neighborhoods higher if they have lower failure rates or capture probabilities, and higher rewards.

The next proposition characterizes the optimal criminal search behavior.

Proposition 1 (Weitzman, 1979). *The optimal criminal behavior is as follows:*

(a) Selection rule: *The criminal inspects neighborhoods in descending order of z_n.*

(b) Stopping rule: *The criminal commits a crime as soon as he finds an unprotected and unpoliced neighborhood; otherwise, he stops and takes the outside option.*

When the neighborhood is neither policed nor protected, the criminal gets r_n which is higher than the reservation index z_n and, by the search rule, higher than the reservation indexes of all remaining unsearched neighborhoods. Hence, the search terminates as soon as the inspected neighborhood is unpoliced and unprotected. Also, although the criminal can return to a previously inspected neighborhood, this decision yields a payoff that is at most equal to the outside option; hence, either the criminal keeps searching or abstains from crime.

Having explained how criminals optimally search for crime targets, we turn to the determination of crime rates. Following recent advancements in the consumer search literature (see, e.g., Choi et al., 2018), we recast our sequential criminal search problem as a discrete choice one. To this end, let u_n denote the *realized payoff* in neighborhood $n \in \mathcal{N}$, where $u_n \in \{-f, r_n - f, 0, r_n\}$. Also, define $w_n := \min\{u_n, z_n\}$ for each $n \in \mathcal{N}$.

Proposition 2. *Given $(u_i, z_i)_{i \in \mathcal{N}}$, a crime is committed in neighborhood i if and only if $w_i > 0$ and $w_i > w_j$ for all other neighborhoods $j \neq i$.*

Proposition 2 is useful in that it provides a straightforward way to compute crime rates κ_n. Indeed, define U_n as the discrete random variable with probability density function (PDF):

$$\mathbb{P}(U_n = u_n) = \begin{cases} (1-p_n)(1-\phi_n) & u_n = r_n; \\ p_n(1-\phi_n) & u_n = r_n - f; \\ (1-p_n)\phi_n & u_n = 0; \\ p_n\phi_n & u_n = -f. \end{cases}$$

Likewise, let $W_n := \min\{U_n, z_n\}$ denote the truncated random variable. Therefore, the *crime rate* κ_n in neighborhood n is given by the probability of the following event:

$$\kappa_n = \mathbb{P}(W_n > 0, W_n > W_j \ \forall j \neq n).$$

The next lemma provides a sharp, yet intuitive, characterization of the crime rates $\kappa_n, n \in \mathcal{N}$.

Lemma 2. *Given indexes $(z_n)_{n \in \mathcal{N}}$, the crime rate in neighborhood n is given by:*

$$\kappa_n = (1-p_n)(1-\phi_n) \prod_{j: z_j \geq z_n} (1 - (1-p_j)(1-\phi_j)). \tag{3}$$

Notice that the expression for the crime rate κ_n has a natural structure. Since the criminal continues searching whenever the neighborhood inspected is either policed or privately protected, κ_n equals the probability that all neighborhoods ranked higher than n are either policed or protected but neighborhood n is not. Consequently, the crime rate in neighborhood n not only depends on its own levels of vigilance ϕ_n and policing p_n, but also on the vigilance and policing levels of all the neighborhoods that are more attractive than n.

5 Optimal policing with criminal search

Having explained how crime rates are determined in each neighborhood, we now turn to the optimal allocation of policing. To this end, let $\boldsymbol{p} \in [0,1]^{\mathcal{N}}$ denote a policing allocation. As previously explained, the crime rate in each neighborhood n (i.e., κ_n in (3)) depends on the criminal selection rule which, in turn, depends on the reservation indexes $\boldsymbol{z}(\boldsymbol{p}) := (z_n(p_n))_{n\in\mathcal{N}}$ in (2). That said, the police agency chooses $\boldsymbol{p}$ to solve:

$$\begin{aligned}
\min \quad & \sum_{n\in\mathcal{N}} \kappa_n(\boldsymbol{p}, \boldsymbol{z}(\boldsymbol{p})) \\
\text{s.t.} \quad & \sum_{n\in\mathcal{N}} p_n \leq \bar{p} \\
& p_n \geq 0 \quad \forall n \in \mathcal{N}
\end{aligned}$$

Because the optimal criminal selection rule is endogenous to the policing allocation, solving the problem above is not straightforward. In principle, a small change in policing can alter the whole distribution of crime across neighborhoods. Our next result shows that although the amount of crime in each neighborhood effectively depends on the criminal search behavior, the total crime rate does not. For some intuition, consider the example below.

Example 1. There are two neighborhoods, 1 and 2. Observe that if the policing allocation encourages criminals to search neighborhood 1 first (i.e., $z_1 > z_2$), then the total crime is:

$$\underbrace{(1-\phi_1)(1-p_1)}_{\text{crime rate in neighborhood 1}} + \underbrace{(1-\phi_2)(1-p_2)(1-(1-\phi_1)(1-p_1))}_{\text{crime rate in neighborhood 2}}.$$

Conversely, if criminals search neighborhood 2 first (i.e., $z_2 > z_1$), the total crime rate is:

$$\underbrace{(1-\phi_2)(1-p_2)}_{\text{crime rate in neighborhood 2}} + \underbrace{(1-\phi_1)(1-p_1)(1-(1-\phi_2)(1-p_2))}_{\text{crime rate in neighborhood 1}}.$$

Notice that the total crime rate — namely, the sum of crime rates in neighborhoods 1 and 2 — is the same in each case. In other words, the total crime rate does not depend on which neighborhood is searched first by criminals. In fact, this feature holds in general. $\diamond$

Proposition 3. *The total crime rate does not depend on the criminal selection rule.*

To understand this result, it is useful to consider a probabilistic interpretation of our model. To this end, let us fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. For each neighborhood $n \in \mathcal{N}$, we define independent Bernoulli random variables $X_n : \Omega \to \{0, 1\}$ with $X_n(\omega) = 1$ if neighborhood n is unprotected and not policed, and $X_n(\omega) = 0$ otherwise. Next, for each n, we define the event $A_n = \{\omega \in \Omega : X_n(\omega) = 1\}$ with probability $\mathbb{P}(A_n) = (1 - \phi_n)(1 - p_n)$. Appendix A.5 shows that the total crime rate can be expressed as:

$$\sum_{n \in \mathcal{N}} \kappa_n(\boldsymbol{p}, \boldsymbol{z}(\boldsymbol{p})) = \mathbb{P}\left(\bigcup_{n \in \mathcal{N}} A_n\right). \tag{4}$$

In other words, the total crime rate can be seen as the probability of a single "success" among a fixed number of heterogeneous trials. Consequently, given policing allocation $\boldsymbol{p}$, the total crime rate is not influenced by the specific search behavior of criminals, since the right-hand side of (4) is not impacted by the criminal selection rule (search order).

Equipped with Proposition 3 and equation (4), we can provide a tractable and closed-form expression for the total crime rate as a function of $\boldsymbol{p} = (p_n)_{n \in \mathcal{N}}$. Indeed, since a crime in neighborhood n "succeeds" with chance $(1 - p_n)(1 - \phi_n)$ and "fails" with complementary chance $p_n(1 - \phi_n) + \phi_n$, it follows that the total crime rate can be rewritten as:

$$\sum_{n \in \mathcal{N}} \kappa_n(\boldsymbol{p}, \boldsymbol{z}(\boldsymbol{p})) = 1 - \mathbb{P}\left(\bigcap_{n \in \mathcal{N}} A_n^c\right) = 1 - \prod_{n \in \mathcal{N}} \mathbb{P}(A_n^c) = 1 - \prod_{n \in \mathcal{N}} ((1 - \phi_n)p_n + \phi_n). \tag{5}$$

Therefore, minimizing the total crime rate is equivalent to *maximizing* the total *deterrence rate* $\prod_{n \in \mathcal{N}} (p_n(1 - \phi_n) + \phi_n)$ — namely, the chance that all neighborhoods searched by criminals are either policed or protected, thereby inducing the criminal to abstain from crime. Because the argmax is invariant to monotone increasing transformations, we can log-transform the total deterrence rate and let the optimal policing allocation $\boldsymbol{p}^*$ solve:

$$\begin{aligned} \max \quad & \sum_{n \in \mathcal{N}} \log(p_n(1 - \phi_n) + \phi_n) \\ \text{s.t.} \quad & \sum_{n \in \mathcal{N}} p_n \leq \bar{p} \\ & p_n \geq 0 \quad \forall n \in \mathcal{N} \end{aligned}$$

Our next result characterizes the optimal policing allocation. Without loss of generality, let us sort neighborhoods according to their failure rates, so that $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq \phi_N < 1$.

Proposition 4. *Let $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq \phi_N < 1$. Define the critical neighborhood as:*

$$n^* = \max \left\{ n \in \mathcal{N} : \frac{\bar{p}}{n} + \frac{1}{n} \sum_{i=1}^{n} \frac{\phi_i}{1-\phi_i} - \frac{\phi_n}{1-\phi_n} > 0 \right\}.$$

Then, the optimal policing allocation is given by:

$$p_n^* = \frac{\bar{p}}{n^*} + \frac{1}{n^*} \sum_{i=1}^{n^*} \frac{\phi_i}{1-\phi_i} - \frac{\phi_n}{1-\phi_n} \quad \forall n \leq n^*,$$

and $p_n^* = 0$ for all neighborhoods $n > n^*$.

The induced optimization problem is well-behaved in that its objective is a strictly concave function, while the constraints form a convex set. Thus, its unique solution can be found using standard nonlinear programming methods; see Appendix A.6. The optimal policing allocation satisfies the familiar "bang-per-buck" principle from consumer theory: the marginal increase in deterrence in neighborhood n from a one-unit increase in policing p_n is the same across all policed neighborhoods $n \leq n^*$; that is, for any pair $n, n' \leq n^*$, the following holds:

$$\frac{1-\phi_n}{p_n^*(1-\phi_n)+\phi_n} = \frac{1-\phi_{n'}}{p_{n'}^*(1-\phi_{n'})+\phi_{n'}} \iff p_n^* + \frac{\phi_n}{1-\phi_n} = p_{n'}^* + \frac{\phi_{n'}}{1-\phi_{n'}}. \tag{6}$$

Hence, the optimal policing intensity in neighborhood n must strictly decrease with its vigilance level ϕ_n to maintain its marginal deterrence at par with other policed neighborhoods, reflecting a *substitution effect* between policing and vigilance.

Along this latter line, Proposition 4 shows that it could be optimal to police only a strict subset of neighborhoods — namely, those with low enough vigilance. In fact, from the definition of n^*, we see that $n^* \geq 1$, i.e., it is always optimal to police at least the lowest vigilance neighborhood. As policing resources $\bar{p}$ increase, more neighborhoods are policed (i.e., n^* weakly increases with $\bar{p}$). Still, since $\bar{p} < 1$, policing all neighborhoods need not occur: using the same definition, we see that it is never optimal to police high-vigilance neighborhoods when the following condition holds:

$$\bar{p} \leq \sum_{n \in \mathcal{N}} \left(\frac{\phi_N}{1-\phi_N} - \frac{\phi_n}{1-\phi_n} \right).$$

This condition is easier to satisfy when: (i) policing resources $\bar{p}$ are scarce; (ii) the number of neighborhoods N is large; and (iii) the *gap* between the highest and lowest vigilance neighborhoods is sizable. For instance, if the lowest vigilance neighborhood has zero vigilance (i.e., $\min_n \phi_n = 0$) and the highest one has at least 50% vigilance (i.e., $\max_n \phi_n \geq 0.5$), then the last policed neighborhood $n^* < N$.[17] Conversely, when there is no vigilance gap, or $\max_n \phi_n = \min_n \phi_n$, then all neighborhoods are policed at the same intensity $\bar{p}/N$; that is, the crime-minimizing allocation is *completely fair* (Persico, 2002): policing resources are distributed equally across all neighborhoods. This condition is also necessary.

Corollary 1. *The optimal policing allocates an equal share of resources to all neighborhoods, i.e., $p_n^* = \bar{p}/N$ for all $n \in \mathcal{N}$ if, and only if, all neighborhoods have the same vigilance level $\phi_n = \phi_k$ for all $n, k \in \mathcal{N}$.*

Thus, the degree of *policing inequality* across neighborhoods owes primarily to the *vigilance inequality* between these ones. Consider two policed neighborhoods, $p_i^*, p_j^* > 0$. Using (6), the degree of policing inequality between neighborhoods i and j is given by

$$|p_i^* - p_j^*| = \frac{|\phi_j - \phi_i|}{(1 - \phi_j)(1 - \phi_i)}.$$

Consequently, when the vigilance gap between i and j increases, policing is more unequal between i and j. In turn, more vigilance inequality leads to fewer policed neighborhoods (n^* falls) and greater policing inequality within policed neighborhoods.[18]

Finally, we discuss in more detail which neighborhoods should be policed and, if so, how much policing each should be allocated. By Proposition 4, the allocation of policing is from bottom to top: low vigilance neighborhoods get priority. Thus, if neighborhood $n > 1$ is policed, then all neighborhoods with lower or equal vigilance should be policed too. Given this, to determine whether n should be policed, the social planner needs to compare a fair split of available resources $\bar{p}/n$ against the average vigilance gap (measured in odds) between n all lower vigilance neighborhoods $i < n$, namely, $\frac{1}{n}\sum_{i=1}^{n}(\frac{\phi_n}{1-\phi_n} - \frac{\phi_i}{1-\phi_i})$. Neighborhood n is policed if and only if the fair split $\bar{p}/n$ exceeds this gap. Thus, neighborhood n^* in Proposition 4 is the last one to satisfy this property. Having determined n^*, the social planner would then allocate a fair share $\bar{p}/n^*$ to all neighborhoods $n \leq n^*$ minus/plus an amount that equals the excess/lack of vigilance of n compared to the average.

All in all, our analysis uncovers that to optimally allocate resources, it is crucial to gather information about the vigilance levels of all relevant neighborhoods. The comparison

[17] In this case, $\sum_n(\frac{\phi_N}{1-\phi_N} - \frac{\phi_n}{1-\phi_n}) > 1 > \bar{p}$, implying that policing all neighborhoods is suboptimal.

[18] Welfare losses from employing a simple and fair allocation are small when the number of neighborhoods is large, since deterring crime is hard when there are too many alternatives to the criminals. However, losses need not be small in the opposite case. For instance, let $\bar{p} = 0.8$ and consider two unequal neighborhoods with $\phi_1 = 0$ and $\phi_2 = 0.5$. The optimal policing entails $p_1^* = \bar{p}$ and $p_2^* = 0$, whereas the fair one allocates $\bar{p}/2$ to each. The respective total crime (5) for the optimal and fair allocation are 0.6 and 0.72. In other words, moving from optimal to fair allocation raises crime by 20%.

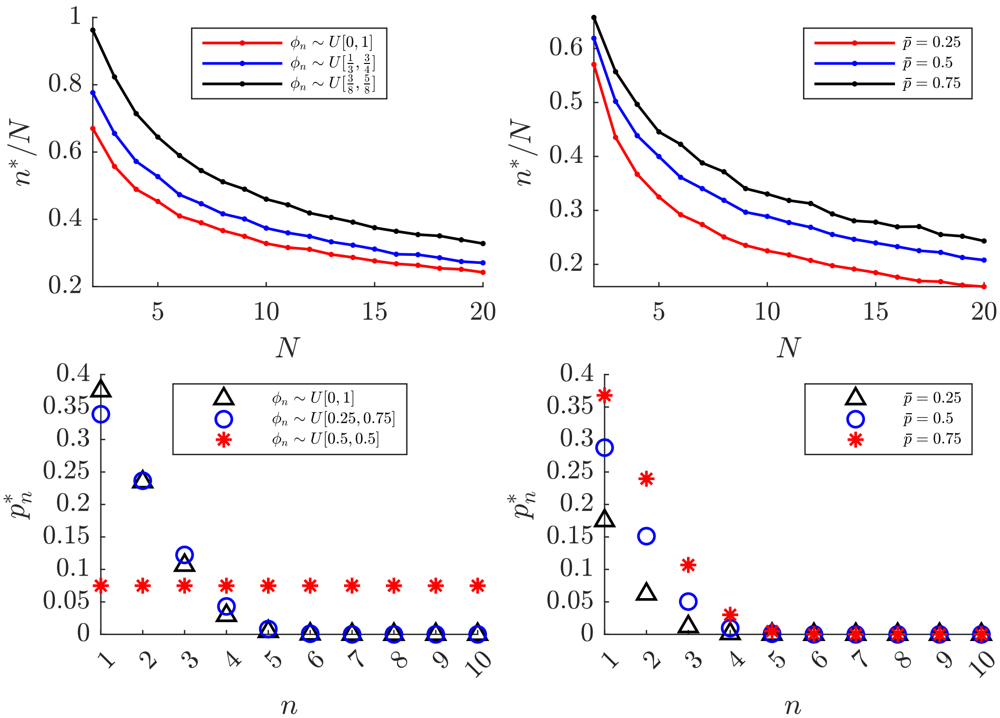

Figure 1: **Policing coverage and intensity with criminal search.** The top panels depict the coverage as a function of the number of neighborhoods, as well as the inequality in vigilance (left panel) and police resources (right panel). The bottom panels illustrate how the policing level in each neighborhood depends on the vigilance inequality across neighborhoods (left panel) and police resources (right panel).

between individual and average vigilance is key to allocating policing to minimize crime. In particular, if neighborhood n is *overprotected* in having vigilance greater than the (conditional) average, then the optimal policing in n is less than the average policing across all policed neighborhoods $\bar{p}/n^*$, while the opposite holds if neighborhood n is *under-protected*.

We illustrate Proposition 4 using numerical simulations of a random crime and policing model. To this end, we first fix both the total number of neighborhoods N and policing resources $\bar{p}$. Next, we generate a random sample of $K = 1,000$ observations as follows. For each observation $k = 1,\ldots,K$, the failure rates ϕ_n^k are drawn independently from a uniform distribution. We then compute the optimal policing allocation n_k^* and $p_{n,k}^*$ for each k. Finally, we plot the sample averages $\frac{1}{K}\sum n_k^*$ and $\frac{1}{K}\sum p_{n,k}^*$, and perform comparative statics. The top panels of Figure 1 depict the fraction of policed neighborhoods, or *coverage*, as a function of N. In particular, the left panel fixes $\bar{p} = 0.75$ and depicts how the coverage depends on the degree of vigilance inequality: as the number of neighborhoods rises, the optimal coverage falls (due to resources being fixed); furthermore, the effect is *amplified* if, in addition, neighborhoods have greater degrees of vigilance inequality. The top right panel illustrates a similar effect but highlights the role of policing resources.

The bottom panels fix $N = 10$ and depict the optimal policing in each neighborhood n (sorted from low to high vigilance). The left one sets $\bar{p} = 0.75$ and illustrates how the optimal policing allocation turns completely fair as vigilance inequality vanishes. The right panel considers $\phi_n \sim U[0, 1]$ and depicts how the optimal policing falls as resources available fall.

6 Optimal policing without criminal search

So far, we have examined the optimal policing allocation when potential criminals inspect neighborhoods sequentially before deciding whether to commit a crime or not. The inspection process can be seen as criminals acquiring information about the neighborhood before taking an action. It is plausible that in some cases this information acquisition process is prohibitively costly for some criminals, differentiating experts from non-experts (Nee and Meenaghan, 2006), or the security measures may be designed to be hidden from criminals (e.g., silent alarm systems). Critically, if learning whether a neighborhood is policed or protected is too costly or technologically infeasible, then potential criminals will base their decisions using their prior knowledge to compute expected payoffs, as in Becker (1968). We now explore this possibility and show that the optimal policing fundamentally changes compared to the one found in §5. Finding this allocation also demands a new approach.

So motivated, we now examine the optimal policing allocation when potential criminals decide whether and where to commit a crime by simply comparing the expected payoff associated with each neighborhood. We maintain the same timing assumption as in §3, i.e., the police agency chooses first the policing level in each neighborhood $\boldsymbol{p} = (p_n)_{n\in\mathcal{N}}$, and then potential criminals best respond to it. We examine subgame perfect equilibria. Notice that the extra choice of when to attempt a crime is now gone, reflecting the absence of search.

We start analyzing the potential criminals' problem. Given policing $\boldsymbol{p}$, potential criminals attempt a crime in the neighborhood that gives the *highest expected criminal profits*:

$$\pi(\boldsymbol{p}) := \max_{n\in\mathcal{N}} r_n(1 - \phi_n) - p_n f.$$

Let us call $\alpha_n(\boldsymbol{p}) \in [0, 1]$ the probability that the criminal attempts a crime in neighborhood n, given $\boldsymbol{p}$, with $\sum_{n\in\mathcal{N}} \alpha_n(\boldsymbol{p}) \leq 1$ (abstaining from crime is also allowed). Optimal criminal behavior requires the following condition to hold:

$$\alpha_n(\boldsymbol{p}) > 0 \quad \Longrightarrow \quad r_n(1 - \phi_n) - p_n f = \pi(\boldsymbol{p}). \tag{7}$$

That is, a crime is attempted in neighborhood n only if n yields the highest expected profits.

The *criminals' best-response correspondence* $BR_C(\boldsymbol{p})$ contains the profiles $(\alpha_n(\boldsymbol{p}))_{n\in\mathcal{N}}$ for which (7) holds. Given $\boldsymbol{p}$, a criminal strategy is deemed *optimal* if it belongs to $BR_C(\boldsymbol{p})$.

The crime rate in neighborhood n is the chance that an attempted crime "succeeds," which happens when n is neither policed nor privately protected, as in §4. Specifically, given an allocation $\boldsymbol{p}$ and an optimal criminal strategy, the crime rate in neighborhood n is $\alpha_n(\boldsymbol{p})(1-\phi_n)(1-p_n)$. The police agency chooses $\boldsymbol{p}$ to solve:

$$\begin{aligned} \min \quad & \sum_{n\in\mathcal{N}} \alpha_n(\boldsymbol{p})(1-\phi_n)(1-p_n) \\ \text{s.t.} \quad & \sum_{n\in\mathcal{N}} p_n \le \bar{p} \\ & p_n \ge 0 \quad \forall n \in \mathcal{N} \end{aligned}$$

This optimization problem is subtle because, given a policing allocation $\boldsymbol{p}$, the criminals' best-response correspondence $BR_C(\boldsymbol{p})$ need not be single-valued, and thus the total crime rate depends on the optimal criminal strategy profile being selected. This imposes a challenge when assessing the performance of different police allocations, which is a fundamental step for finding optimal policies. To address this issue, we next impose a natural equilibrium selection, or a mild restriction on how an optimal strategy $\alpha_n(\boldsymbol{p})$ changes as $\boldsymbol{p}$ changes.

Consider two policing allocations $\boldsymbol{p}$ and $\tilde{\boldsymbol{p}}$, with $\boldsymbol{p} \neq \tilde{\boldsymbol{p}}$, and suppose that the neighborhoods' sets $\{n : r_n(1-\phi_n) - p_n f = \pi(\boldsymbol{p})\}$ and $\{n : r_n(1-\phi_n) - \tilde{p}_n f = \pi(\tilde{\boldsymbol{p}})\}$ coincide. Then, we select optimal criminal strategies $\alpha_n(\boldsymbol{p})$ and $\alpha_n(\tilde{\boldsymbol{p}})$ so that $\alpha_n(\boldsymbol{p}) = \alpha_n(\tilde{\boldsymbol{p}})$ for all n. In other words, if two allocations leave the criminals indifferent among the same set of (most attractive) neighborhoods, then criminals' optimal strategy is the same under any of these allocations, i.e., the criminals' behavior remains unchanged. Henceforth, we keep this restriction, which allows us to establish a natural necessary condition for the optimality of $\boldsymbol{p}$.

Lemma 3. *Suppose $\boldsymbol{p}$ minimizes total crime. Then, $p_n > 0 \implies r_n(1-\phi_n) - p_n f = \pi(\boldsymbol{p})$.*

Intuitively, Lemma 3 states that policing neighborhoods that are unattractive to criminals is suboptimal for the police agency, as these resources could help reduce crime had they been allocated to more attractive neighborhoods for them. Hence, in equilibrium, policed neighborhoods must be those that offer the highest expected profits to criminals.

Next, without loss of generality, we sort neighborhoods according to their expected criminal rewards, or *rents*: $r_1(1-\phi_1) \le \cdots \le r_N(1-\phi_N)$. Similar to §5, we will show that the optimal policing allocation has a cutoff structure. Indeed, given this neighborhood sorting and Lemma 3, policed neighborhoods ($p_n > 0$) must have higher rents $r_n(1-\phi_n)$ than unpoliced ones ($p_n = 0$).[19] This suggests that, in equilibrium, policed neighborhoods n are those above a *critical neighborhood* n^* and provide the highest expected profits to criminals. We can then recast the police agency problem as choosing a *policy* consisting of a critical neighborhood n^*, ensuring that feasibility, neighborhood sorting, and the necessary conditions for police optimality are met. We say policy $n^* \in \mathcal{N}$ is *implementable* if there exists a policing allocation $\boldsymbol{p} = (p_n)_{n\in\mathcal{N}}$ such that the following three conditions hold:

(i) $p_n > 0 \Rightarrow r_n(1-\phi_n) - p_n f = \pi(\boldsymbol{p})$;

(ii) $r_n(1-\phi_n) - p_n f = \pi(\boldsymbol{p})$ if and only if $n \geq n^*$;

(iii) $\sum_{n\in\mathcal{N}} p_n = \bar{p}$ with $p_n \geq 0$.

This reformulation of the police's problem leads to a clean and tractable description of equilibria, as the set of implementable policies has a simple characterization. Define $\zeta : \mathcal{N} \to \mathbb{R}$ as:

$$\zeta(n) := \frac{-\bar{p}f + \sum_{k\geq n} r_k(1-\phi_k)}{N-n+1}. \tag{8}$$

We say that $\hat{n}$ is a *local max* of ζ if there exists $\delta \geq 1$ such that $\zeta(\hat{n}) \geq \zeta(n)$ for all $n \in \mathcal{N}$ within (euclidean) distance δ of $\hat{n}$. Our next result fully characterizes the set of implementable policies.

Proposition 5. *Policy $n^* \in \mathcal{N}$ is implementable if and only if n^* is a local max of ζ.*

For some intuition, suppose that policy n^* is implementable. Then, $\zeta(n^*)$ determines the *equilibrium profits* to criminals. Indeed, let $\pi^* := \pi(\boldsymbol{p})$. Then, $r_n(1-\phi_n) - p_n f = \pi^*$ for all $n \geq n^*$. Solving for p_n yields $p_n = (r_n(1-\phi_n) - \pi^*)/f$. Using the binding resource constraint $\sum_n p_n = \bar{p}$, we get $\pi^* = \zeta(n^*)$. Now, to understand why n^* must be a local max of ζ, we exploit the following lemma, which is related to the criminals' incentives.

Lemma 4. *Let $n \in \mathcal{N}$ with $n < N$. The following statements are equivalent:*

(i) $\zeta(n+1) \geq \zeta(n)$;

(ii) $\zeta(n+1) \geq r_n(1-\phi_n)$;

(iii) $\zeta(n) \geq r_n(1-\phi_n)$.

When n^* is implementable, $\zeta(n^*)$ equals the equilibrium criminal profits. This means that attempting a crime in unpoliced neighborhoods must leave criminals worse off, i.e.,

[19] In effect, if $p_n > p_k = 0$ and $r_n(1-\phi_n) \leq r_k(1-\phi_k)$, then $r_n(1-\phi_n) - p_n f < \pi(\boldsymbol{p})$, and thus allocation $\boldsymbol{p}$ cannot be optimal, according to Lemma 3.

$\zeta(n^*) \geq r_n(1-\phi_n)$ for all $n < n^*$. By our neighborhood sorting, this reduces to $\zeta(n^*) \geq r_{n^*-1}(1-\phi_{n^*-1})$, or $\zeta(n^*) \geq \zeta(n^*-1)$ by Lemma 4. In addition, as previously mentioned, implementability requires $p_n = (r_n(1-\phi_n) - \pi^*)/f$ for $n \geq n^*$. Since $p_n \geq 0$, the latter condition is equivalent to $\zeta(n^*) \leq r_{n^*}(1-\phi_{n^*})$, or $\zeta(n^*) \geq \zeta(n^*+1)$ by Lemma 4. In sum, n^* is a local max of ζ. (The only if direction is similar and proved in the appendix.)

Our next lemma puts a constraint on the shape that $\zeta(\cdot)$ can take, allowing us to sharpen further the characterization of the set of implementable policies.

Lemma 5. *Let $n \in \mathcal{N}$ with $1 < n < N$. Then, $\zeta(n+1)-\zeta(n) \geq 0 \implies \zeta(n)-\zeta(n-1) \geq 0$.*

Proof: Consider $n \in \mathcal{N}$ with $1 < n < N$. If $\zeta(n+1) \geq \zeta(n)$ then $\zeta(n) \geq r_n(1-\phi_n)$, by Lemma 4. Thus, $\zeta(n) \geq r_{n-1}(1-\phi_{n-1})$, since $r_n(1-\phi_n) \geq r_{n-1}(1-\phi_{n-1})$, by rent sorting. Consequently, $\zeta(n) \geq \zeta(n-1)$, again by Lemma 4. □

It follows that $\zeta(\cdot)$ is either monotone, or hump-shaped. For instance, suppose that $\zeta(N) \geq \zeta(N-1)$. Then, by Lemma 5, we have that $\zeta(N) \geq \zeta(N-1) \geq \cdots \geq \zeta(1)$, i.e., ζ is monotone increasing. Conversely, if ζ eventually decreases, then it continues to do so. Consequently, policy n^* is implementable if and only if $n^* \in \arg\max_{n\in\mathcal{N}} \zeta(n)$; see Lemma A.1 in Appendix A.10. This allows us to show that all implementable policies lead to the same equilibrium outcome (Lemma A.2). Therefore, the optimal policing allocation is uniquely determined.

Proposition 6. *Let $r_1(1-\phi_1) \leq \cdots \leq r_N(1-\phi_N)$. Define the critical neighborhood as:*

$$n^* = \min\left\{n \in \mathcal{N} : \frac{\bar{p}}{N-n+1} + \frac{1}{N-n+1}\sum_{k=n}^{N}\left(\frac{r_n(1-\phi_n) - r_k(1-\phi_k)}{f}\right) > 0\right\}.$$

Then, the optimal policing allocation is given by:

$$p_n^* = \frac{\bar{p}}{N-n^*+1} + \frac{1}{N-n^*+1}\sum_{k=n^*}^{N}\left(\frac{r_n(1-\phi_n) - r_k(1-\phi_k)}{f}\right) \quad \forall n \geq n^*,$$

and $p_n^* = 0$ for all neighborhoods $n < n^*$.

The logic is the following (the formal proof is in Appendix A.10). Using the observations preceding the proposition, one can show that if $\max_n \zeta(n) > \zeta(N)$ then policy n^* obeys $n^* = \min\{n \in \mathcal{N} : \zeta(n) - \zeta(n+1) > 0\}$, i.e., n^* is the first instance at which ζ strictly decreases. Then, using (8), n^* can be written as in the statement of the proposition. Conversely, if $\max_n \zeta(n) = \zeta(N)$ then ζ must be monotone increasing (Lemma 5), and so $n^* = N$. Clearly, n^* rises as resources $\bar{p}$ rise; hence, one can find thresholds $\bar{p}_\ell$ and $\bar{p}_h$ such that policing all neighborhoods is optimal when $\bar{p} \geq \bar{p}_h$ (large budget), while policing only N is optimal when $\bar{p} \leq \bar{p}_\ell$ (tight budget).[20] As for the optimal policing intensity, rather than equalizing the "bang-per-buck" across policed neighborhoods, as in Proposition 4, the optimal policing equalizes the expected criminal profits within policed neighborhoods, namely, $r_n(1-\phi_n) - p_n^* f = \zeta(n^*)$ for all $n \geq n^*$, which highlights the different types of trade-offs that the police agency faces, depending on the criminals' operation practices (search versus no search).

Proposition 6 shows interesting qualitative differences compared to the case in which criminals search for the best target (Proposition 4). First, while vigilance inequality is key to allocating policing when there is criminal search, here it is the inequality in rents, or *rent inequality*, that shapes the optimal allocation of police resources. Indeed, the optimal policing allocation prioritize high rent neighborhoods; in particular, the highest rent neighborhood ($n = N$) is always policed. Also, the allocation of resources among high rent neighborhoods is from top to bottom, namely, if neighborhood n is policed then all neighborhoods $k > n$ are policed too. So, to determine whether neighborhood n should be policed, the social planner compares a fair split of resources among all neighborhoods with higher rents than n, i.e., $\bar{p}/(N-n+1)$, with the average *rent gap* between n and the rest of the neighborhoods with higher rents than n, $\frac{1}{N-n+1}\sum_{k\geq n}\left(\frac{r_k(1-\phi_k)-r_n(1-\phi_n)}{f}\right)$. Neighborhood n is policed if the fair split exceeds this gap. Hence, n^* is the last neighborhood (from high to low rents) to satisfy this property. After n^* is found, the optimal policing allocates a fair $\bar{p}/(N-n^*+1)$ to all policed neighborhoods $n \geq n^*$ plus (resp. minus) an amount that reflects the excess (resp. lack) of rents of n compared to the average within policed neighborhoods. Neighborhoods with higher rents than the average receive more than a fair share of the police resources.

Second, when criminals search for the best target, there is a *substitution* between vigilance and policing because criminals are highly likely to pass on high vigilance neighborhoods after the inspection stage. In contrast, when search is absent, criminals are drawn to high rent neighborhoods, even if these ones have high vigilance. This means that assigning more police resources to high vigilance neighborhoods could be part of an optimal policing strategy, reflecting possible *complementarities* between vigilance and policing.

Finally, the simple policy of policing all neighborhoods at a common intensity is now optimal when all neighborhoods have equal rents, not necessarily equal vigilance.

Corollary 2. *The optimal policing allocates an equal share of resources to all neighborhoods, i.e., $p_n^* = \bar{p}/N$ for all $n \in \mathcal{N}$ if, and only if, all neighborhoods offer the same expected rewards to criminals, i.e., $r_n(1-\phi_n) = r_k(1-\phi_k)$ for all $n, k \in \mathcal{N}$.*

Thus, if neighborhoods have the same vigilance (say $\phi_n = \phi_0$ for all n) and rewards ($r_n = r_0$

[20] These thresholds are $\bar{p}_h := \sum_{k=1}^{N}[r_k(1-\phi_k) - r_1(1-\phi_1)]/f$ and $\bar{p}_\ell := [r_N(1-\phi_N) - r_{N-1}(1-\phi_{N-1})]/f$.

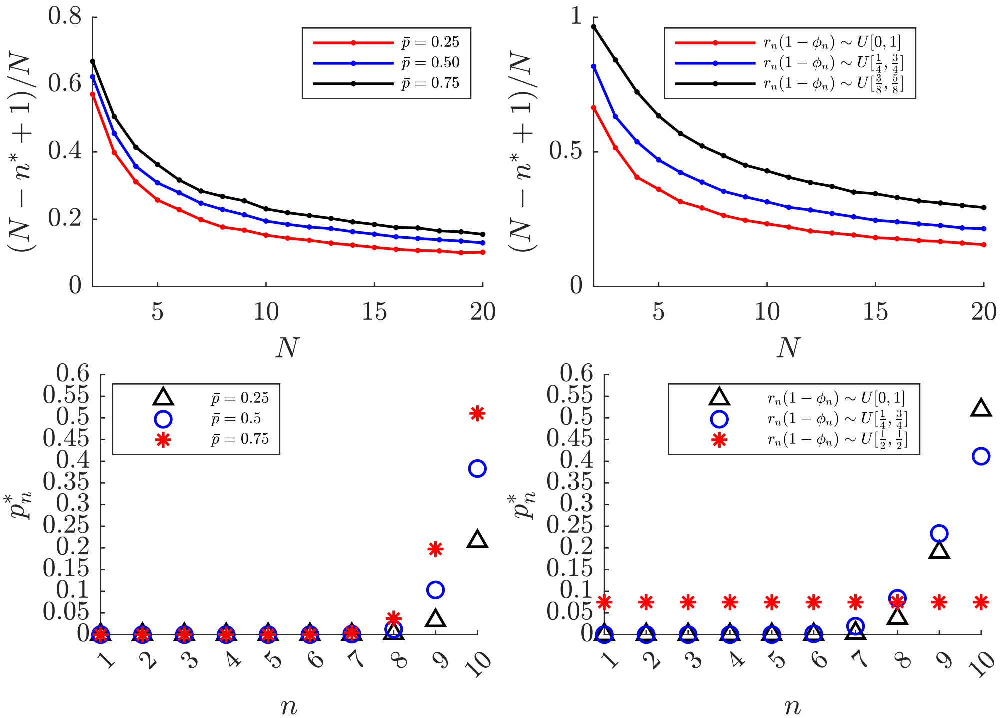

Figure 2: **Policing coverage and intensity without criminal search.** The top panels illustrate the coverage as a function of the number of neighborhoods, the police resources (left panel), and the inequality in rents across neighborhoods (right panel). The bottom panels depict the policing levels across neighborhoods and how it depends on the amount of resources (left panel) and the inequality in rents (right panel).

for all n), then splitting police resources evenly across neighborhoods is optimal, regardless of whether criminals search or not, by Corollaries 1–2. In particular, the optimal allocation does not leverage information on the vigilance (ϕ_0) or rewards (r_0) levels.

Finally, to close this section, we illustrate our results in Figure 2. Concretely, we fix the parameters $(N, \bar{p}, f)$ and construct a random sample of $K = 1,000$ observations. For each $k = 1, \dots, K$, the rents $r_n^k(1-\phi_n^k)$ are drawn independently from a uniform distribution. We then compute the optimal allocation n_k^* and $p_{n,k}^*$ for each k, and plot the sample averages on the vertical axis. In all panels, we consider $f = 0.25$.

The top panels of Figure 2 depict the fraction of policed neighborhoods $(N - n^* + 1)/N$ (coverage) as a function of N. We examine how rent inequality and police resources impact the policing coverage. In particular, the left panel fixes the degree of rent inequality between neighborhoods (uniform between 0 and 1) and depicts how the policing coverage depends on the police resources $\bar{p}$: as the number of neighborhoods rises, the policed coverage falls; however, this can be counteracted by increasing resources $\bar{p}$. The top right panel sets $\bar{p} = 0.75$ and illustrates how rent inequality affects coverage: with no rent inequality, there is full coverage (Corollary 2), yet this one decreases as rent inequality increases.

The bottom panels consider 10 neighborhoods, ordered from low to high rents, and depict the optimal policing in each of them. The bottom left panel fixes the degree of rent inequality between neighborhoods (uniform between 0 and 1) and examines the effect of policing resources. The optimal policing (Proposition 6) rises as $\bar{p}$ rises. The bottom right panel fixes the resource level $\bar{p} = 0.75$ and illustrates how the optimal policing allocation becomes fair as rent inequality vanishes across neighborhoods.

7 Concluding Remarks

Understanding how criminals choose a target among many alternatives is crucial to designing and implementing better crime reduction policies, especially if police agencies face tight budgetary constraints. In this paper, we examine how to allocate scarce police resources across potential crime targets to minimize the overall crime rate. Our analysis accounts for the potential search behavior of criminals, who may expend resources before attempting a crime to ensure success without being caught. We also investigate the counterpart to this scenario, in which these expenditures may be too costly for criminals, reflecting limited expertise, or technological constraints.

Our paper indicates that understanding the factors that determine the criminals' *modus operandi*, as well as specific inequality measures of neighborhoods are central to improving the allocation of police resources to decrease crime. If criminals search before attempting a crime, then the optimal policing allocation depends on the *vigilance inequality* across neighborhoods. This allocation prioritizes low vigilance neighborhoods, providing them resources proportionate to the gap between their vigilance and the average vigilance across policed neighborhoods. Furthermore, the equitable policy of splitting resources evenly across neighborhoods is optimal if and only if all neighborhoods have the same vigilance level.

In contrast, if criminals do not search before attempting a crime, then the optimal police allocation reflects the *rent inequality* across neighborhoods, which is a byproduct of both vigilance and the rewards of each neighborhood. Hence, splitting police resources evenly across neighborhoods is optimal when all neighborhoods offer the same expected rewards to criminals; otherwise, neighborhoods associated with high expected rewards are given priority and are also assigned more police resources. Altogether, our analysis suggests that in addition to gathering data on relevant neighborhood characteristics, it is equally important to collect information on the criminals' operation methods. In fact, policing plays a key role in either substituting or potentially complementing vigilance depending on this consideration.

We conclude with directions for future research. First, to isolate the police allocation trade-off, our framework assumes exogenous vigilance expenditures. Thus, our results are more appropriate to inform policy in the short-run, where these expenditures are likely to be constant. A natural next step is to introduce a vigilance adjustment process by which households actively invest in private security in response to crime. With criminal search, households have incentives to underinvest in vigilance, as they would anticipate greater police protection; in contrast, without criminal search, households may wish to overinvest in vigilance to decrease their attractiveness to criminals. Second, in practice, countries vary by their degree of law enforcement centralization. In the US, for example, neighborhoods are usually policed by different police agencies and their decisions need not be coordinated; thus, extending the framework to accommodate potential strategic interactions between police agencies could be a fruitful exercise.[21] Finally, from an empirical perspective, it would be interesting to use our theoretical results to analyze whether the disparities in policing observed in practice (e.g., Chen et al., 2023) can be justified on efficiency grounds (crime minimization), as well as to evaluate and quantify possible misallocation channels.

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A Omitted Proofs

A.1 Proof of Lemma 1

We solve for z_n in the case where the criminal takes z_n and continues search unless they succeed without getting captured, namely $z_n \in (0, r_n)$. Then, equation (1) is reduced to

$$z_n = -c + r_n(1-\phi_n)(1-p_n) + z_n[1-(1-\phi_n)(1-p_n)].$$

Solving for z_n yields (2). Next, since $c > 0$, it follows that $z_n < r_n$. Moreover, Assumption 1 implies $-c + r_n(1-\phi_n)(1-p_n) > 0$, and thus $z_n > 0$. Altogether, $z_n \in (0, r_n)$. $\square$

A.2 Proof of Proposition 1

By Weitzman (1979) (*Pandora's rule*), the criminal searches in descending order of z_n and terminates the search whenever the maximum sampled payoff exceeds the reservation index z_n of every neighborhood not searched yet. Without loss of generality, let us relabel neighborhoods so that $z_1 \geq z_2 \geq z_3 \geq \cdots \geq z_n$. By Lemma 1, $z_n > 0$ for all n, implying that the criminal finds it optimal to search, as his outside option $u_0 = 0$.

First, we show that if neighborhood n is inspected, then the maximum sampled payoff, i.e. $\max\{u_0, u_1, \ldots, u_{n-1}\}$ must be equal to zero, where $u_i \in \{-f, r_i - f, 0, r_i\}$ denotes the realized payoff in neighborhood i. We argue by contradiction: Suppose the maximum sampled payoff prior to inspecting n is strictly positive.[22] Then $u_i = r_i$ for some neighborhood i inspected prior to n. This implies that, at that moment, $0 = u_0 < u_i$ and $z_i < u_i = r_i$. But then Pandora's rule implies that search must have terminated at that moment, since the maximum sampled payoff is at least r_i which is strictly greater than z_n for all $n \geq i$. Thus, neighborhood n could have not been inspected. We conclude that $\max\{u_0, u_1, \ldots, u_{n-1}\} = 0$.

Next, we show that if neighborhood n is inspected, then the search terminates if and only if n is both unprotected and not policed. If n is inspected then $\max\{u_0, u_1, \ldots, u_{n-1}\} = 0$. Now, if n is unprotected and not policed, then $u_n = r_n > z_i$ for all $i \geq n$ and $u_n > \max\{u_0, u_1, \ldots, u_{n-1}\}$. Thus, by Pandora's rule, the search terminates. Conversely, if n is either protected or policed, then $u_n \leq 0$ and so $\max\{u_0, u_1, \ldots, u_n\} = 0$ which is less than $z_i > 0$ for $i > n$. Thus, the search must continue, again by Pandora's rule.

Finally, if $\max\{u_0, u_1, \ldots, u_N\} = 0$, i.e., all neighborhoods were either policed or protected, then recalling a previously inspected neighborhood is no greater than taking the outside option; hence, the criminal is deterred from crime. □

A.3 Proof of Proposition 2

($\Rightarrow$) We prove that if $w_i > 0$ and $w_i > w_j$, then a crime is committed in neighborhood i. We leverage Proposition 1 throughout the proof. Therefore, if $w_i \equiv \min\{u_i, z_i\} > 0$ then $z_i > 0$; thus, the criminal searches at least one neighborhood. Likewise, $u_i > 0$ implies that the criminal finds it optimal to commit a crime in i, conditional on inspecting i. Hence, it suffices to show that if, in addition, $w_i > w_j$, the crime will be committed in neighborhood i. Next, we separate into two cases.

(a) Suppose $z_j < u_j$. Then $w_j = \min\{u_j, z_j\} = z_j$, and since $w_i > w_j$ and $w_i = \min\{u_i, z_i\}$, it follows that $z_i > z_j$ and $u_i > z_j$. This says that the criminal inspects neighborhood i before j and would have no incentives to visit neighborhood j after inspecting i.

(b) Suppose $z_j > u_j$ so that $w_j = u_j$. Similarly, since $w_i > w_j$, then $z_i > u_j$ and $u_i > u_j$. This says that even if neighborhood j is inspected before i, the criminal has incentives to keep searching ($z_i > u_j$) or to return to a previously inspected neighborhood ($u_i > u_j$). In any case, committing a crime in neighborhood j is suboptimal.

[22] If the maximum sampled payoff is strictly negative, then n will clearly be inspected, as $z_n > 0$.

($\Leftarrow$) Now we prove the converse. Next, we'll show that if a crime is committed in neighborhood i, then $w_i > w_j$ and $w_i > 0$. We prove the contrapositive: if $w_i \leq w_j$ or $w_i \leq 0$, then it is suboptimal to commit a crime in i. First, since $z_i \in (0, r_i)$ (Lemma 1), if $w_i \leq 0$ then $u_i \leq 0$. Thus, the criminal will not commit a crime in i, because the realized value u_i is dominated by the outside option ($u_0 = 0$). Second, if $w_i \leq w_j$ then it is suboptimal to commit a crime in i, following the same logic of cases (a) and (b) above. Altogether, if $w_i \leq w_j$ or $w_i \leq 0$, then the criminal will never commit a crime in neighborhood i. □

A.4 Proof of Lemma 2

Then $\mathbb{P}(W_n = w_n)$ has the following PDF:

$$\mathbb{P}(W_n = w_n) = \begin{cases} (1-p_n)(1-\phi_n) & w_n = z_n; \\ p_n(1-\phi_n) & w_n = r_n - f; \\ (1-p_n)\phi_n & w_n = 0; \\ p_n\phi_n & w_n = -f. \end{cases}$$

The crime rate in neighborhood n is given by $\kappa_n = \mathbb{P}(W_n > W_j, W_n > 0, \forall j \neq n)$. Note that $\mathbb{P}(W_n > W_j, W_n > 0, \forall j \neq n)) = \mathbb{P}(W_n > W_j, \forall j \neq n | W_n > 0)\mathbb{P}(W_n > 0)$ and $\mathbb{P}(W_n > 0) = \mathbb{P}(W_n = z_n) = (1-p_n)(1-\phi_n)$. Also, by independence,

$$\mathbb{P}(W_n > W_j, \forall j \neq n | W_n > 0) = \prod_{j \neq n} \mathbb{P}(W_n > W_j | W_n > 0).$$

The problem reduces to finding $\mathbb{P}(W_n > W_j | W_n > 0)$. First, we find each individual probability for the 4 possible values that W_j could take. Since $z_j > 0 > r_j - f$ for all j:

$$\mathbb{P}(W_n > -f | W_n > 0) = \mathbb{P}(W_n > r_j - f | W_n > 0) = \mathbb{P}(W_n > 0 | W_n > 0) = 1.$$

If $W_j = z_j$, the probability depends on the order between z_j and z_n. In particular, for $z_n > z_j$, $\mathbb{P}(W_n > z_j | W_n = z_n) = 1$. On the other hand, for $z_n \leq z_j$, $\mathbb{P}(W_n > z_j | W_n = z_n) = 0$. Define $\mathcal{N}_n^* := \{j \in \mathcal{N} : z_j \geq z_n\}$ as the set of neighborhoods ranked above z_n. Observe that

$$\begin{aligned} \mathbb{P}(W_n > W_j, \forall j \neq n | W_n > 0) &= \mathbb{P}(W_n > W_j, \forall j \notin \mathcal{N}_n^* | W_n > 0)\mathbb{P}(W_n > W_j, \forall j \in \mathcal{N}_n^* | W_n > 0) \\ &= \prod_{j \notin \mathcal{N}_n^*} \mathbb{P}(W_n > W_j | W_n > 0) \prod_{j \in \mathcal{N}_n^*} \mathbb{P}(W_n > W_j | W_n > 0). \end{aligned}$$

We now separate into cases. First consider the case in which $j \notin \mathcal{N}_n^*$. For $z_j < z_n$:

$$\mathbb{P}(W_n > W_j | W_n > 0) = 1 p_j \phi_j + 1 p_j (1 - \phi_j) + 1(1 - p_j)\phi_j + 1(1 - p_j)(1 - \phi_j) = 1.$$

Next, we examine $\mathbb{P}(W_n > W_j | W_n > 0)$ for $j \in \mathcal{N}_n^*$. Here, we have

$$\begin{aligned}\mathbb{P}(W_n > W_j | W_n > 0) &= 1 p_j \phi_j + 1 p_j (1 - \phi_j) + 1(1 - p_j)\phi_j + 0(1 - p_j)(1 - \phi_j) \\ &= 1 - (1 - p_j)(1 - \phi_j).\end{aligned}$$

Thus, the crime rate in i is given by:

$$\begin{aligned}\mathbb{P}(W_n > W_j, \forall j \neq n | W_n > 0) &= \mathbb{P}(W_n > W_j, \forall j \notin \mathcal{N}_n^* | W_n > 0)\mathbb{P}(W_n > W_j, \forall j \in \mathcal{N}_n^* | W_n > 0) \\ &= \prod_{j \notin \mathcal{N}_n^*} 1 \prod_{j \in \mathcal{N}_n^*} (1 - (1 - p_j)(1 - \phi_j)) = \prod_{j \in \mathcal{N}_n^*} (1 - (1 - p_j)(1 - \phi_j)).\end{aligned}$$

Altogether, it follows that

$$\begin{aligned}\mathbb{P}(W_n > W_j, W_n > 0, \forall j \neq n) &= \mathbb{P}(W_n > W_j, \forall j \neq n | W_n > 0)\mathbb{P}(W_n > 0) \\ &= (1 - p_n)(1 - \phi_n) \prod_{j \in \mathcal{N}_n^*} (1 - (1 - p_j)(1 - \phi_j))\end{aligned}$$

This completes the proof. □

A.5 Proof of Proposition 3

Fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. For each neighborhood $n = 1, \ldots, N$, we define the random variable $X_n : \Omega \to \{0, 1\}$ with $X_n(\omega) = 1$ if neighborhood n is unprotected and not policed, and $X_n(\omega) = 0$ otherwise. By assumption, the collection of random variables $(X_n)_{n=1}^N$ are independent from one another. Next, for each n, we define the event $A_n = \{\omega \in \Omega : X_n(\omega) = 1\}$, and we observe that $\mathbb{P}(A_n) = (1 - \phi_n)(1 - p_n)$.

We will show that for any search order, the crime rate is unchanged. To this end, consider the *simple* search order, in which the criminal first inspects neighborhood 1, then neighborhood 2, etc. Given this simple search, we define $\mathcal{K}(X_1, \ldots, X_N) \in \{0, 1\}$ as the realized crime rate. By Lemma 2, this function is given by:

$$\mathcal{K}(X_1, \ldots, X_N) := X_1 + X_2(1 - X_1) + \cdots + X_N \prod_{i=1}^{N-1} (1 - X_i).$$

Thus, the total crime rate is determined by $\mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)]$, i.e.:

$$\sum_{n=1}^{N}\kappa_n = \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)].$$

The next lemma shows that, for the simple search order, the total crime rate is equal to the probability of having a successful crime among N trials. Specifically,

Lemma 6. *For the simple search order,* $\mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] = \mathbb{P}(\bigcup_{n=1}^{N} A_n)$ *for all* $N \geq 2$*.*

Proof: We prove the claim by induction. Consider the base case $N = 2$. Then, $\mathcal{K}(X_1, X_2) = X_1 + X_2(1 - X_1)$, and so:

$$\begin{aligned}
\mathbb{E}[\mathcal{K}(X_1, X_2)] &= \mathbb{E}(X_1) + \mathbb{E}(X_2)(1 - \mathbb{E}(X_1)) \\
&= \mathbb{E}(X_1) + \mathbb{E}(X_2) - \mathbb{E}(X_1)\mathbb{E}(X_2) \\
&= \mathbb{P}(A_1) + \mathbb{P}(A_2) - \mathbb{P}(A_1)\mathbb{P}(A_2) \\
&= \mathbb{P}(A_1) + \mathbb{P}(A_2) - \mathbb{P}(A_1 \cap A_2) \\
&= \mathbb{P}(A_1 \cup A_2).
\end{aligned}$$

Next, suppose the formula holds for $N > 2$. We'll show that this implies that the formula must hold for $N + 1$. To this end, let $\hat{A}_N := \bigcup_{n=1}^{N} A_n$ and notice that $\hat{A}_N$ is independent of A_{N+1}. Thus,

$$\begin{aligned}
\mathbb{P}(\hat{A}_n \cup A_{N+1}) &= \mathbb{P}(\hat{A}_N) + \mathbb{P}(A_{N+1}) - \mathbb{P}(\hat{A}_N \cap A_{N+1}) \\
&= \mathbb{P}(\hat{A}_N) + \mathbb{P}(A_{N+1}) - \mathbb{P}(\hat{A}_N)\mathbb{P}(A_{N+1}) \\
&= \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] + \mathbb{E}(X_{N+1})(1 - \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)]),
\end{aligned}$$

where we used independence to get the second equality, and the inductive hypothesis to get the third one. Now, notice that,

$$1 - \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] = 1 - \mathbb{P}\left(\bigcup_{n=1}^{N} A_n\right) = \mathbb{P}\left((\bigcup_{n=1}^{N} A_n)^c\right) = \mathbb{P}\left(\bigcap_{n=1}^{N} A_n^c\right) = \prod_{n=1}^{N}\mathbb{P}(A_n^c).$$

Since $\mathbb{P}(A_n^c) = 1 - \mathbb{P}(A_n) = 1 - (1-\phi_n)(1-p_n)$, it follows that:

$$
\begin{aligned}
\mathbb{P}(\hat{A}_n \cup A_{N+1}) &= \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] + \mathbb{E}(X_{N+1})(1 - \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)]) \\
&= \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] + \mathbb{P}(A_{N+1}) \prod_{n=1}^{N} \mathbb{P}(A_n^c) \\
&= \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] + (1-\phi_{N+1})(1-p_{N+1}) \prod_{n=1}^{N} (1 - (1-\phi_n)(1-p_n)) \\
&= \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)] + \kappa_{N+1} = \sum_{n=1}^{N+1} \kappa_n \\
&= \mathbb{E}[\mathcal{K}(X_1,\ldots,X_{N+1})].
\end{aligned}
$$

In other words, $\mathbb{E}[\mathcal{K}(X_1,\ldots,X_{N+1})] = \mathbb{P}(\bigcup_{n=1}^{N+1} A_n)$, and the claim is proved. □

By Lemma 6, the total crime rate for the simple search order obeys

$$
\sum_{n=1}^{N} \kappa_n = \mathbb{P}\left(\bigcup_{n=1}^{N} A_n \right),
$$

but notice that the right hand side of this expression must be independent of the search order because the union of sets is obviously commutative. More precisely, consider any permutation $\sigma : \{1,\ldots,N\} \to \{1,\ldots,N\}$. Then,

$$
\mathbb{E}[\mathcal{K}(X_{\sigma(1)},\ldots,X_{\sigma(N)})] = \mathbb{P}\left(\bigcup_{n=1}^{N} A_{\sigma(n)} \right) = \mathbb{P}\left(\bigcup_{n=1}^{N} A_n \right) = \mathbb{E}[\mathcal{K}(X_1,\ldots,X_N)].
$$

Since a search order can be seen as a permutation over a set of alternatives, we conclude that the total crime rate is independent of $\sigma(\cdot)$. □

A.6 Proof of Proposition 4

Notice that the objective function is strictly concave, while the constraint set is convex. Thus, the Karush-Kuhn-Tucker (KKT) conditions will effectively identify the optimal policing allocation $\boldsymbol{p}^*$. The Lagrangian of this problem is given by:

$$
\mathcal{L}(\boldsymbol{p}) = \sum_{n\in\mathcal{N}} \log(p_n(1-\phi_n) + \phi_n) - \lambda \left(\sum_{n\in\mathcal{N}} p_n - \bar{p} \right) + \sum_{n\in\mathcal{N}} \mu_n p_n,
$$

where $\lambda \geq 0$ and $\mu_n \geq 0$ denote the Lagrange multipliers. The first-order conditions are:

$$\frac{1-\phi_n}{p_n(1-\phi_n)+\phi_n} - \lambda + \mu_n = 0 \quad \forall n \in \mathcal{N}. \tag{9}$$

We'll show that there exists non-negative multipliers $(\lambda, (\mu_n)_{n\in\mathcal{N}})$ such that the profile $((p_n^*)_{n\in\mathcal{N}}, (\mu_n)_{n\in\mathcal{N}}, \lambda)$ satisfies the KKT conditions. To this end, consider n^* as in the statement of the proposition. First, since $p_n^* > 0$ for all $n \leq n^*$, we set $\mu_n = 0$ for $n \leq n^*$ to satisfy complementary slackness. Second, consider $\lambda > 0$ as follows:

$$\lambda^{-1} = \frac{\bar{p}}{n^*} + \frac{1}{n^*}\sum_{i=1}^{n^*} \frac{\phi_i}{1-\phi_i}. \tag{10}$$

Then, it is easy to see that, given λ in (10), p_n^* solves the FOC (9) for all $n \leq n^*$.

Next, we use (9) to find μ_n for $n > n^*$ and verify that their values are non-negative. Using (10) for λ and $p_n^* = 0$ we get:

$$\mu_n = \left(\frac{\bar{p}}{n^*} + \frac{1}{n^*}\sum_{i=1}^{n^*} \frac{\phi_i}{1-\phi_i}\right)^{-1} - \frac{1-\phi_n}{\phi_n} \quad \forall n > n^*$$

Hence, $\mu_n \geq 0$ if and only if,

$$\frac{\bar{p}}{n^*} + \frac{1}{n^*}\sum_{i=1}^{n^*} \frac{\phi_i}{1-\phi_i} - \frac{\phi_n}{1-\phi_n} \leq 0 \quad \forall n > n^*,$$

which is, indeed, the case, given the definition of n^*.

Finally, we verify that the resource constraint holds. Since $\lambda > 0$, the resource constraint binds at the optimum, which is also satisfied because:

$$\sum_{n\in\mathcal{N}} p_n^* = \sum_{n=1}^{n^*} p_n^* = \bar{p} + \sum_{n=1}^{n^*} \frac{\phi_n}{1-\phi_n} - \sum_{n=1}^{n^*} \frac{\phi_n}{1-\phi_n} = \bar{p}.$$

Altogether, the profile $((p_n^*)_{n\in\mathcal{N}}, (\mu_n)_{n\in\mathcal{N}}, \lambda)$ satisfies the KKT conditions, and thus $(p_n^*)_{n\in\mathcal{N}}$ is an optimal policing allocation. □

A.7 Proof of Lemma 3

We prove the contrapositive. Suppose that neighborhood n is policed ($p_n > 0$) and $r_n(1-\phi_n) - p_n f < \pi(\boldsymbol{p})$. Condition (7) implies that $\alpha_n(\boldsymbol{p}) = 0$. We'll show that there exists another policing allocation $\tilde{\boldsymbol{p}}$ that leads to strictly less crime than $\boldsymbol{p}$ does; thus, $\boldsymbol{p}$ cannot minimize crime. To this end, consider the set K of all neighborhoods k satisfying $r_k(1-\phi_k) - p_k f = \pi(\boldsymbol{p})$. Let $\tilde{p}_n = p_n - \epsilon$ for small $\epsilon > 0$, and

$$\tilde{p}_k = p_k + \frac{\epsilon}{|K|} \quad \forall k \in K.$$

For the remaining neighborhoods, let $\tilde{p}_k = p_n$ for $k' \notin K \cup \{n\}$. Clearly, $\tilde{\boldsymbol{p}}$ satisfies the resource constraint since $\boldsymbol{p}$ does.

Next, under policing $\tilde{\boldsymbol{p}}$, the expected criminal profits are $r_k(1-\phi_k) - \tilde{p}_k f = \pi(\boldsymbol{p}) - \epsilon/|K|$ for all $k \in K$. Moreover, since for all $k' \notin K \cup \{n\}$ we have $\pi(\boldsymbol{p}) > r_{k'}(1-\phi_{k'}) - p_{k'} f$, it follows that we can choose $\epsilon > 0$ such that $\epsilon \leq p_n$ and also:

$$\begin{aligned} \frac{\epsilon}{|K|} &< \pi(\boldsymbol{p}) - \left(\max_{k \notin K \cup \{n\}} r_k(1-\phi_k) - p_k f \right); \\ \frac{\epsilon}{|K|} + \epsilon f &< \pi(\boldsymbol{p}) - (r_n(1-\phi_n) - p_n f). \end{aligned}$$

This ensures that, for any ϵ satisfying the conditions above, we have that

$$\pi(\tilde{\boldsymbol{p}}) = \pi(\boldsymbol{p}) - \frac{\epsilon}{|K|} > r_k(1-\phi_k) - \tilde{p}_k f \quad \forall k \notin K,$$

implying that $\alpha_n(\boldsymbol{p}) = \alpha_n(\tilde{\boldsymbol{p}})$ for all n. Moreover, since $\tilde{p}_k > p_k$ for all $k \in K$:

$$\sum_{n \in \mathcal{N}} \alpha_n(\tilde{\boldsymbol{p}})(1-\phi_n)(1-\tilde{p}_n) < \sum_{n \in \mathcal{N}} \alpha_n(\boldsymbol{p})(1-\phi_n)(1-p_n).$$

Consequently $\boldsymbol{p}$ cannot be optimal. Altogether, we conclude that, at the optimum, $p_n > 0$ implies $r_n(1-\phi_n) - p_n f = \pi(\boldsymbol{p})$. $\square$

A.8 Proof of Lemma 4

(i) $\iff$ (ii). By definition,

$$\begin{aligned}
\zeta(n+1)-\zeta(n) &= \frac{-\bar{p}f+\sum_{k\geq n+1} r_k(1-\phi_k)}{N-n}+\frac{\bar{p}f-\sum_{k\geq n} r_k(1-\phi_k)}{N-n+1} \\
&= \frac{-\bar{p}f+\sum_{k\geq n+1} r_k(1-\phi_k)}{N-n}+\frac{\bar{p}f-r_n(1-\phi_n)-\sum_{k\geq n+1} r_k(1-\phi_k)}{N-n+1} \\
&= \left(-\bar{p}f+\sum_{k\geq n+1} r_k(1-\phi_k)\right)\left(\frac{1}{N-n}-\frac{1}{N-n+1}\right)-\frac{r_n(1-\phi_n)}{N-n+1} \\
&= \left(\frac{-\bar{p}f+\sum_{k\geq n+1} r_k(1-\phi_k)}{N-n}-r_n(1-\phi_n)\right)\frac{1}{N-n+1} \\
&= (\zeta(n+1)-r_n(1-\phi_n))\frac{1}{N-n+1}
\end{aligned}$$

Consequently, $\zeta(n+1)\geq\zeta(n)$ if and only if $\zeta(n+1)\geq r_n(1-\phi_n)$.

(i) $\iff$ (iii). We proceed analogously but expressing the difference $\zeta(n+1)-\zeta(n)$ in terms of $\zeta(n)$:

$$\begin{aligned}
\zeta(n+1)-\zeta(n) &= \frac{-\bar{p}f+\sum_{k\geq n+1} r_k(1-\phi_k)}{N-n}+\frac{\bar{p}f-\sum_{k\geq n} r_k(1-\phi_k)}{N-n+1} \\
&= \frac{-\bar{p}f+\sum_{k\geq n} r_k(1-\phi_k)-r_n(1-\phi_n)}{N-n}+\frac{\bar{p}f-\sum_{k\geq n} r_k(1-\phi_k)}{N-n+1} \\
&= \left(-\bar{p}f+\sum_{k\geq n} r_k(1-\phi_k)\right)\left(\frac{1}{N-n}-\frac{1}{N-n+1}\right)-\frac{r_n(1-\phi_n)}{N-n} \\
&= \left(\frac{-\bar{p}f+\sum_{k\geq n} r_k(1-\phi_k)}{N-n+1}-r_n(1-\phi_n)\right)\frac{1}{N-n} \\
&= (\zeta(n)-r_n(1-\phi_n))\frac{1}{N-n}
\end{aligned}$$

Thus, $\zeta(n+1)\geq\zeta(n)$ if and only if $\zeta(n)\geq r_n(1-\phi_n)$. □

A.9 Proof of Proposition 5

The "if" part is proved on the main text. We now prove the "only if" direction. Suppose that n^* is a local max of ζ. Without loss, let $n^*\notin\{1,N\}$ (the cases $n^*=1$ or $n^*=N$ are analogous).[23] Then, it follows that $\zeta(n^*)\geq\zeta(n^*-1)$ and $\zeta(n^*)\geq\zeta(n^*+1)$. By Lemma 4, these inequalities can be expressed as $\zeta(n^*)\geq r_{n^*-1}(1-\phi_{n^*-1})$ and $\zeta(n^*)\leq r_{n^*}(1-\phi_{n^*})$.

[23] To implement neighborhood $n=1$, we only need to satisfy $\zeta(1)\leq r_1(1-\phi_1)$, while to implement neighborhood $n=N$, we only require $\zeta(N)\geq r_{N-1}(1-\phi_{N-1})$.

Let $p_n^* = (r_n(1-\phi_n) - \zeta(n^*))/f$ for $n \geq n^*$ and $p_n^* = 0$ for $n < n^*$. Then, $p_n^* \geq 0$ for all n, and given the definition of ζ:

$$\sum_n p_n^* f = \sum_{n \geq n^*} r_n(1-\phi_n) - (N - n^* + 1)\zeta(n^*) = \bar{p}f,$$

and so the resource constraint is satisfied. Next, we show that neighborhoods $n^* \geq n$ deliver the highest expected profits to criminals. Indeed, $r_n(1-p_n^*) - p_n^* f = \zeta(n^*)$ for all $n \geq n^*$; also, $\zeta(n^*) \geq r_{n^*-1}(1-\phi_{n^*-1}) = \max_{n<n^*} r_n(1-\phi_n)$. Finally, we notice that all neighborhoods with expected profits strictly less than $\zeta(n^*)$ have zero policing $p_n^* = 0$. Altogether, n^* is implementable. $\square$

A.10 Proof of Proposition 6

To prove this result, we leverage two lemmas.

Lemma A.1. *Consider $\zeta(\cdot)$ in (8). If $n^* \in \mathcal{N}$ locally maximizes ζ, then $\zeta(n^*) = \max_{n \in \mathcal{N}} \zeta(n)$.*

Proof: We'll show that any local max of ζ must be a global max. This is trivially true if $\arg\max_n \zeta(n)$ is single-valued. So suppose that n_1^* and n_2^* locally maximize ζ, with $n_2^* > n_1^*$. We will show that $\zeta(n_1^*) = \zeta(n_2^*)$. On the one hand, since n_1^* locally maximizes ζ, it follows that $\zeta(n_1^*) \geq \zeta(n_1^* + 1)$; thus, Lemma 5 implies that ζ must weakly decrease for $n \geq n_1^*$: $\zeta(n_1^*) \geq \zeta(n_1^*+1) \geq \cdots \geq \zeta(N)$. As a result, $\zeta(n_1^*) \geq \zeta(n_2^*)$. On the other hand, n_2^* also locally maximizes ζ, and so $\zeta(n_2^*) \geq \zeta(n_2^* - 1)$; therefore, Lemma 5 implies that ζ must weakly increase for $n \leq n_2^*$: $\zeta(1) \leq \zeta(2) \leq \cdots \leq \zeta(n_2^*)$. Consequently, $\zeta(n_1^*) \leq \zeta(n_2^*)$. Altogether, $\zeta(n_1^*) = \zeta(n_2^*)$. Finally, since a global max is, in particular, a local max, it follows that any local maximizer n^* is a global maximizer, i.e., $\zeta(n^*) = \max_{n \in \mathcal{N}} \zeta(n)$. $\square$

Lemma A.2. *Let $\boldsymbol{p}_{\boldsymbol{n}^*}^*$ denote the optimal policing allocating when $n^* \in \mathcal{N}$ is implemented. Then, if policies n_1^* and n_2^* are both implementable, then $\boldsymbol{p}_{\boldsymbol{n}_1^*}^* = \boldsymbol{p}_{\boldsymbol{n}_2^*}^*$.*

Proof: We'll show that any implementable policy leads to the same optimal policing allocation. The statement is trivially true if $\arg\max_n \zeta(n)$ is single-valued. So suppose that n_1^* and n_2^* are both implementable, with $n_1^* < n_2^*$. By Proposition 5, this means that both n_1^* and n_2^* locally maximize ζ, and thus $\zeta(n_1^*) = \zeta(n_2^*) = \max_n \zeta(n)$, by Lemma A.1. Moreover, by the same logic given in the proof of Lemma A.1, it easily follows that $\zeta(k) = \max_n \zeta(n)$ for all $k \in \{n_1^*, \ldots, n_2^*\}$. That is, all neighborhoods between n_1^* and n_2^* are also implementable.

Next, we'll show that $\max_n \zeta(n) = r_k(1-\phi_k)$ for all $k \in \{n_1^*, \ldots, n_2^* - 1\}$. To see this, let $k \in \{n_1^*, \ldots, n_2^* - 1\}$. Then, $\zeta(k+1) = \zeta(k)$, and so $\zeta(k+1) = \zeta(k) = r_k(1-\phi_k)$, by Lemma 4. Hence, $\max_n \zeta(n) = r_k(1-\phi_k)$ since k is implementable.

Having established these facts, suppose policy n_1^* is implemented. In the main text, we show that $\zeta(n_1^*)$ equals the equilibrium profits for criminals. Thus, the optimal policing $\boldsymbol{p}^*_{\boldsymbol{n_1^*}}$ entails allocating resources so that $r_k(1-\phi_k)-p^*_{k,n_1^*}f=\zeta(n_1^*)$ for all $k \geq n_1^*$ and $p^*_{k,n_1^*}=0$ for $k<n_1^*$. However, because $\zeta(n^*)=\zeta(k)=r_k(1-\phi_k)$ for all $k \in \{n_1^*,\ldots,n_2^*-1\}$, it follows that $p^*_{k,n_1^*}=0$ for all $k \in \{n_1^*,\ldots,n_2^*-1\}$. As a result, $\boldsymbol{p}^*_{\boldsymbol{n_1^*}}=\boldsymbol{p}^*_{\boldsymbol{n_2^*}}$ since $\zeta(n_2^*)=\zeta(n_1^*)$. $\square$

Proof of Proposition 6: As previously argued, Lemma 5 indicates that ζ in (8) is either monotone or hump-shaped. As a result, any local max is a global max (Lemma A.1). In addition, Lemma A.2 indicates that all implementable policies (or global maximizers of ζ) lead to the same optimal policing allocation; in particular, for any implementable policy k^*, we have $\boldsymbol{p}^*_{\boldsymbol{k^*}}=\boldsymbol{p}^*_{\boldsymbol{n^*}}$, where $n^*=\max\{n \in \mathcal{N} : n \text{ is implementable}\}$. This implies that, without loss of generality, the optimal policing allocation can be written as: $p_n^*=0$ for all $n<n^*$ and $p_n^*=(r_n(1-\phi_n)-\zeta(n^*))/f$ for all $n \geq n^*$. Using (8), the latter transforms into:

$$p_n^* = \frac{\bar{p}}{N-n^*+1} + \frac{1}{N-n^*+1}\sum_{k=n^*}^{N}\left(\frac{r_n(1-\phi_n)-r_k(1-\phi_k)}{f}\right).$$

Finally, notice that if $n^*<N$ then n^* can be expressed as $n^*=\min\{n \in \mathcal{N}\setminus\{N\} : \zeta(n)-\zeta(n+1)>0\}$. Also, the proof of Lemma 4 indicates that $\zeta(n)-\zeta(n+1)=[r_n(1-\phi_n)-\zeta(n)]/(N-n)$ for $n<N$. Thus, using (8) again, n^* can be written as

$$n^* = \min\left\{n \in \mathcal{N}\setminus\{N\} : \frac{\bar{p}}{N-n+1} + \frac{1}{N-n+1}\sum_{k=n}^{N}\left(\frac{r_n(1-\phi_n)-r_k(1-\phi_k)}{f}\right) > 0\right\}.$$

Conversely, if the set above is empty, then n^* cannot be strictly less than N; therefore, $n^*=N$. Altogether, we can write n^* as in the statement of the proposition. $\square$